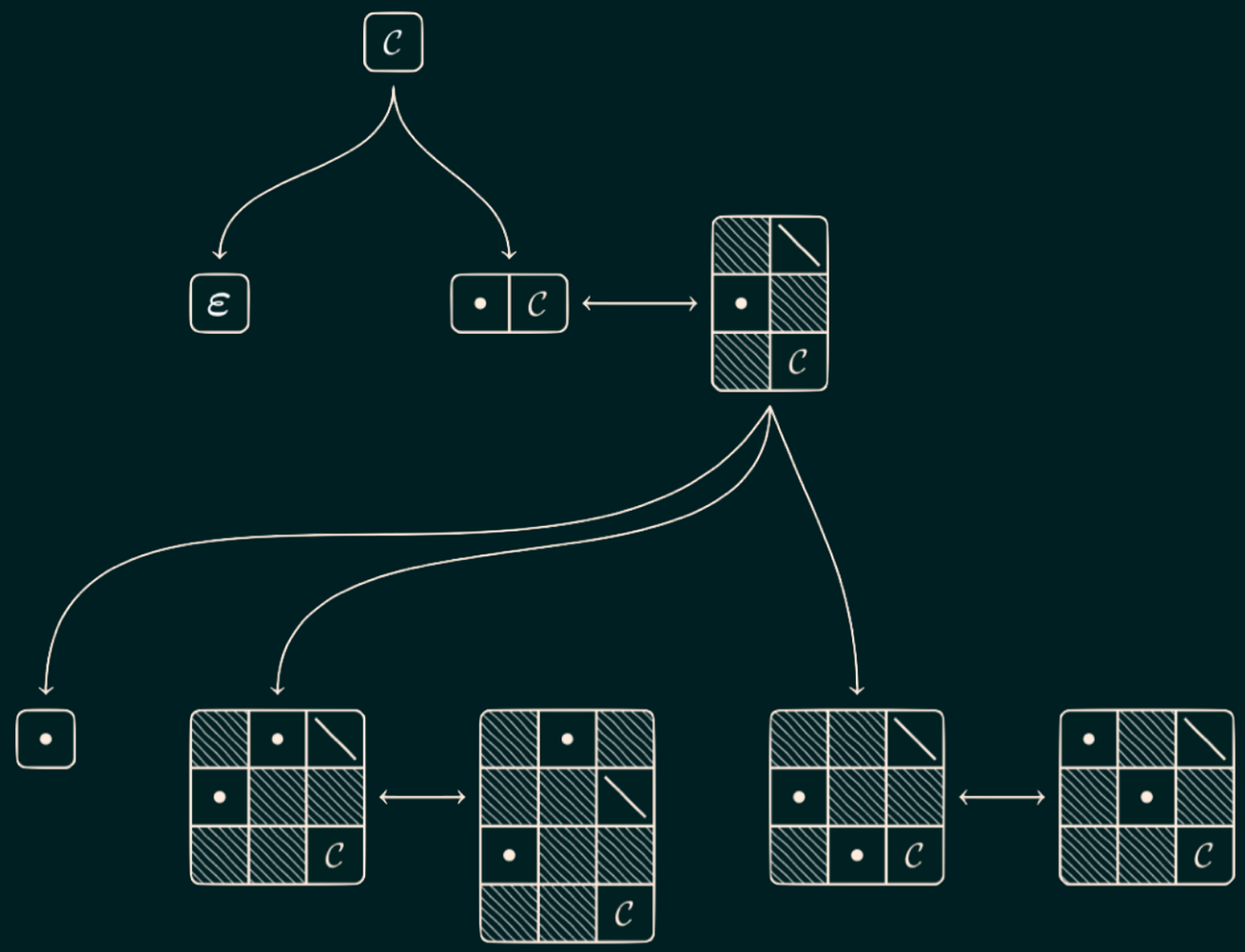
Combinatorial Exploration

an algorithmic framework for enumeration

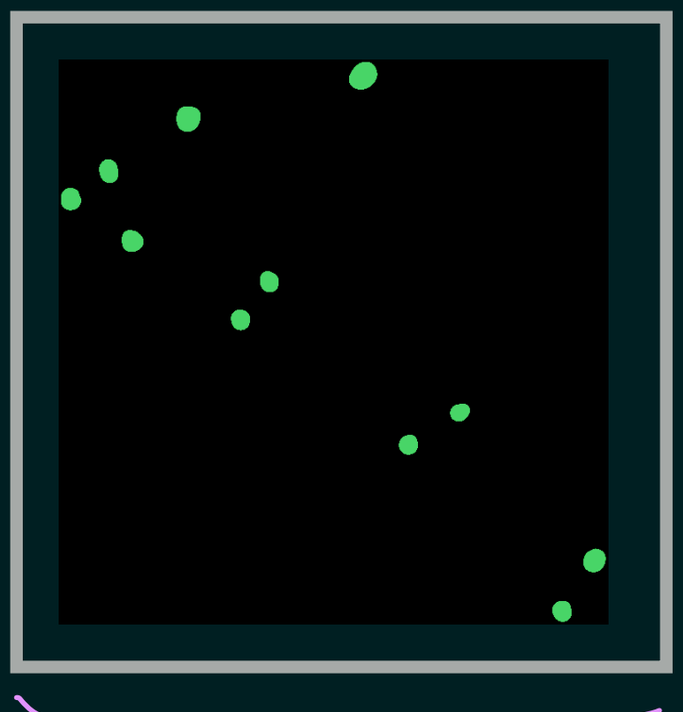
Jay Pantone
Marquette University

with:
Michael Albert
Christian Bean
Anders Claesson
Émile Nadeau
Henning Ulfarsson

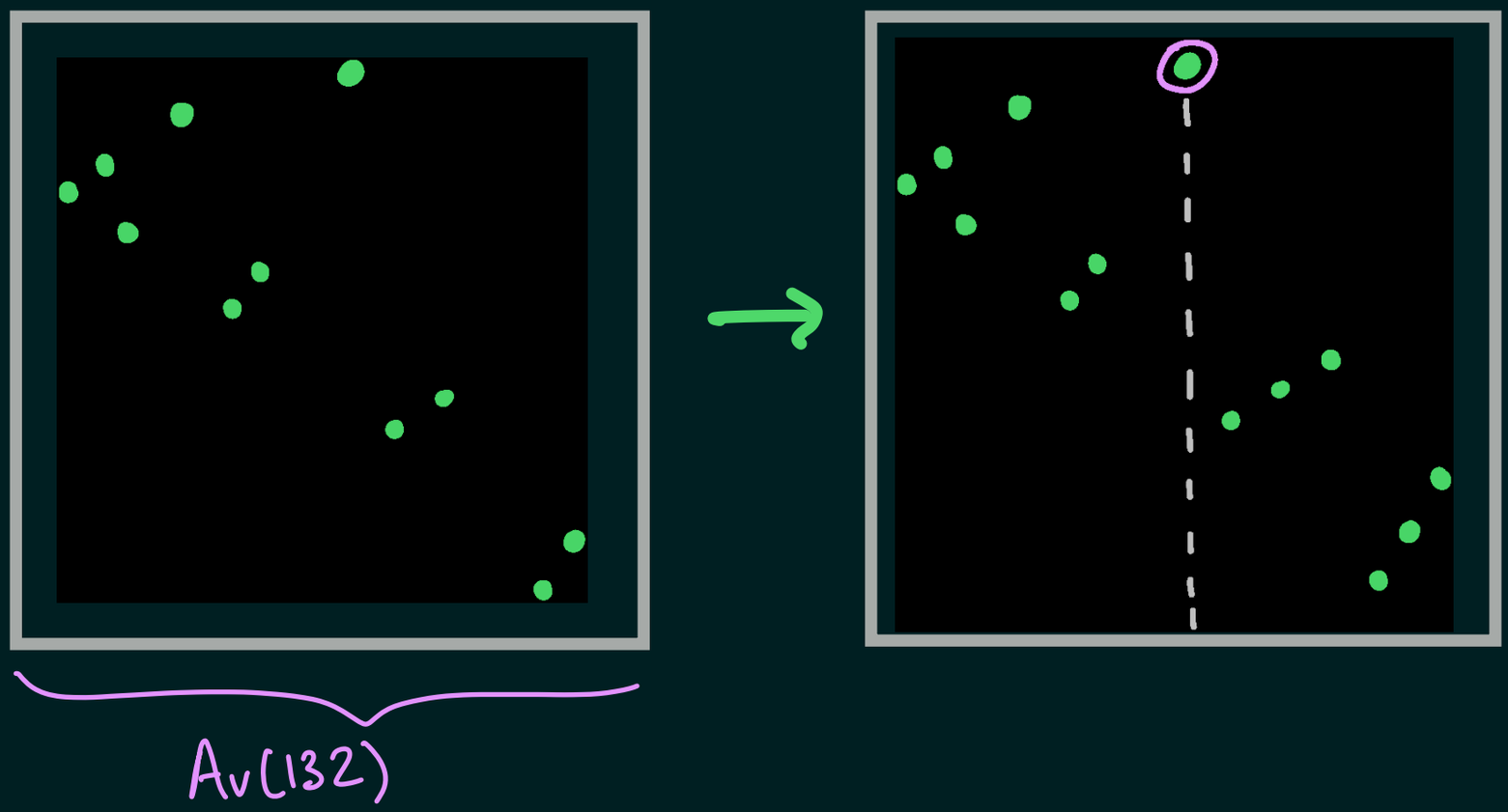


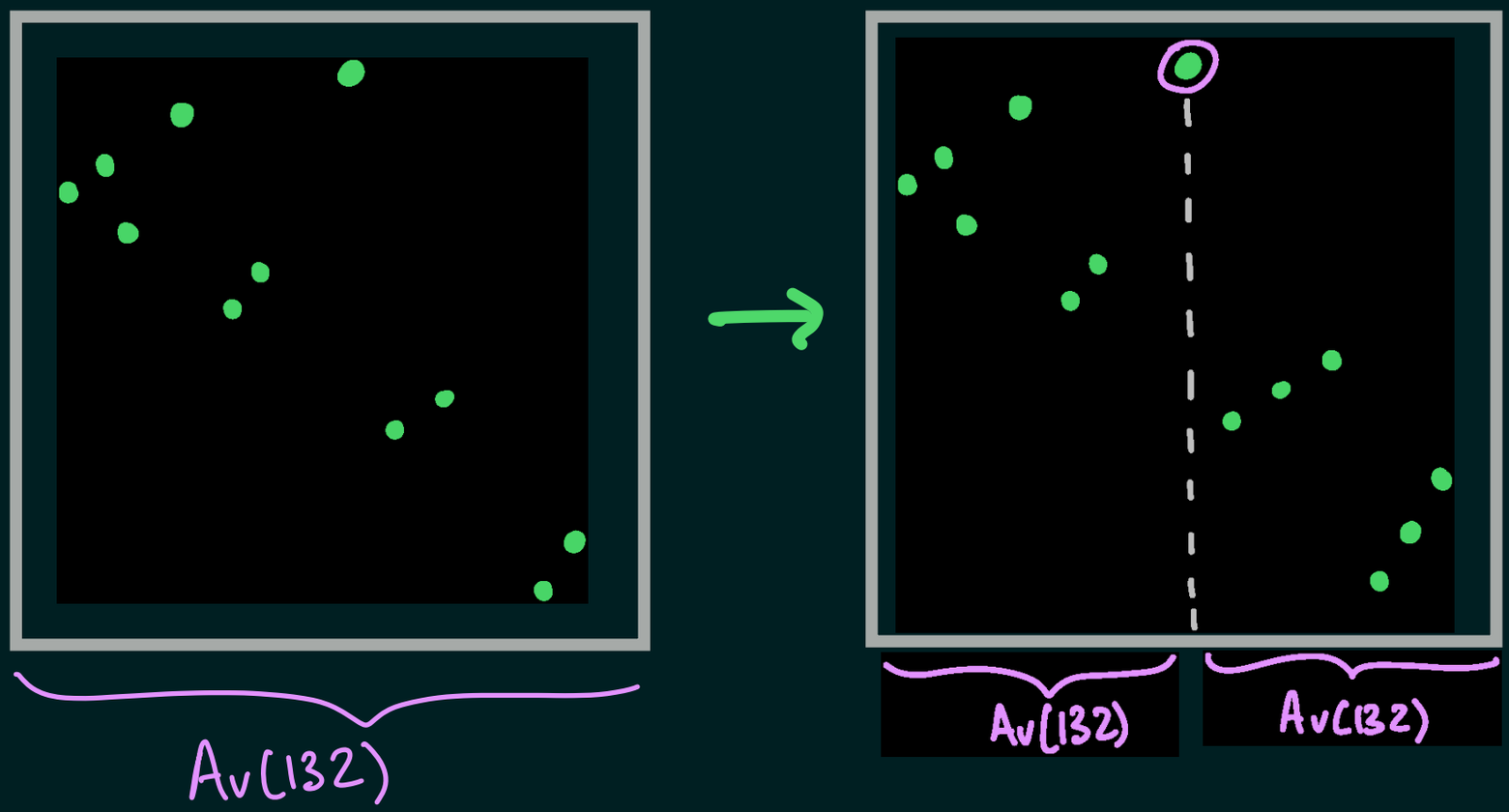
Extra references:

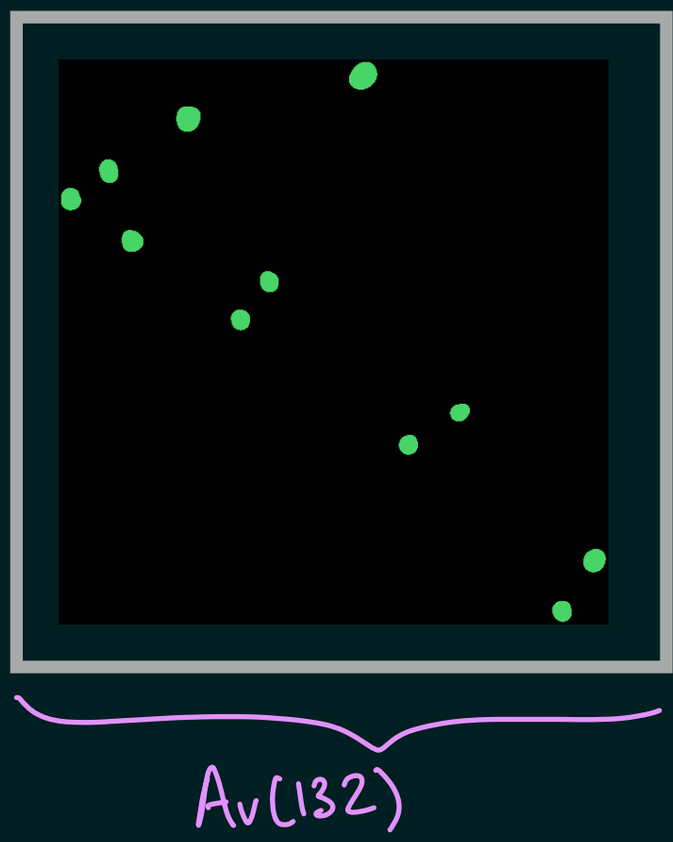
- preprint: https://arxiv.org/abs/2202.07715
- Longer talk specifically about Combinatorial Exploration (domain-agnostic): https://vimeo.com/687277242
- PermPAL: https://permpal.com/

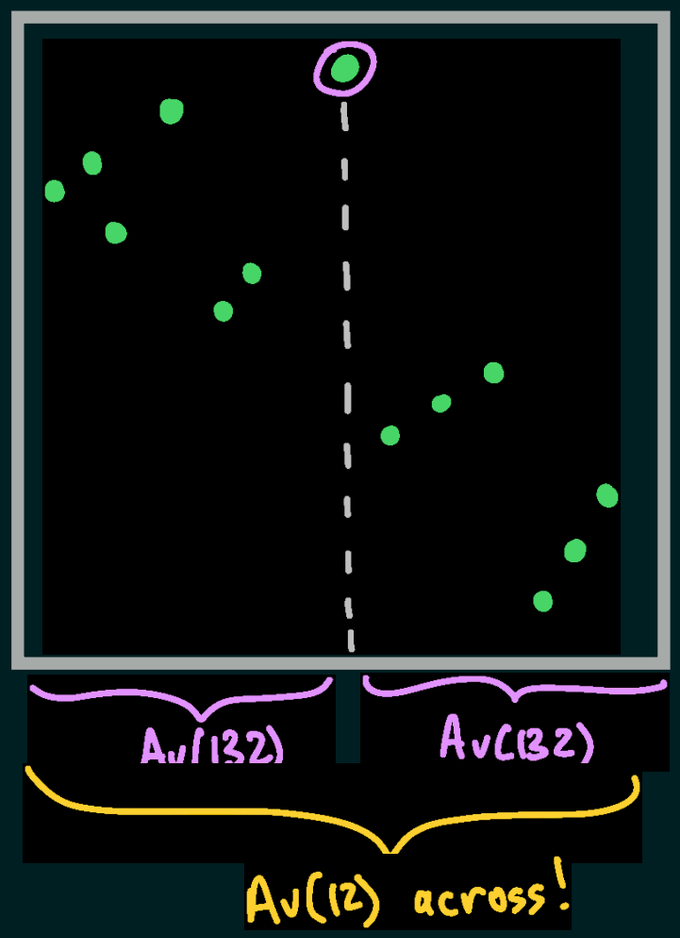


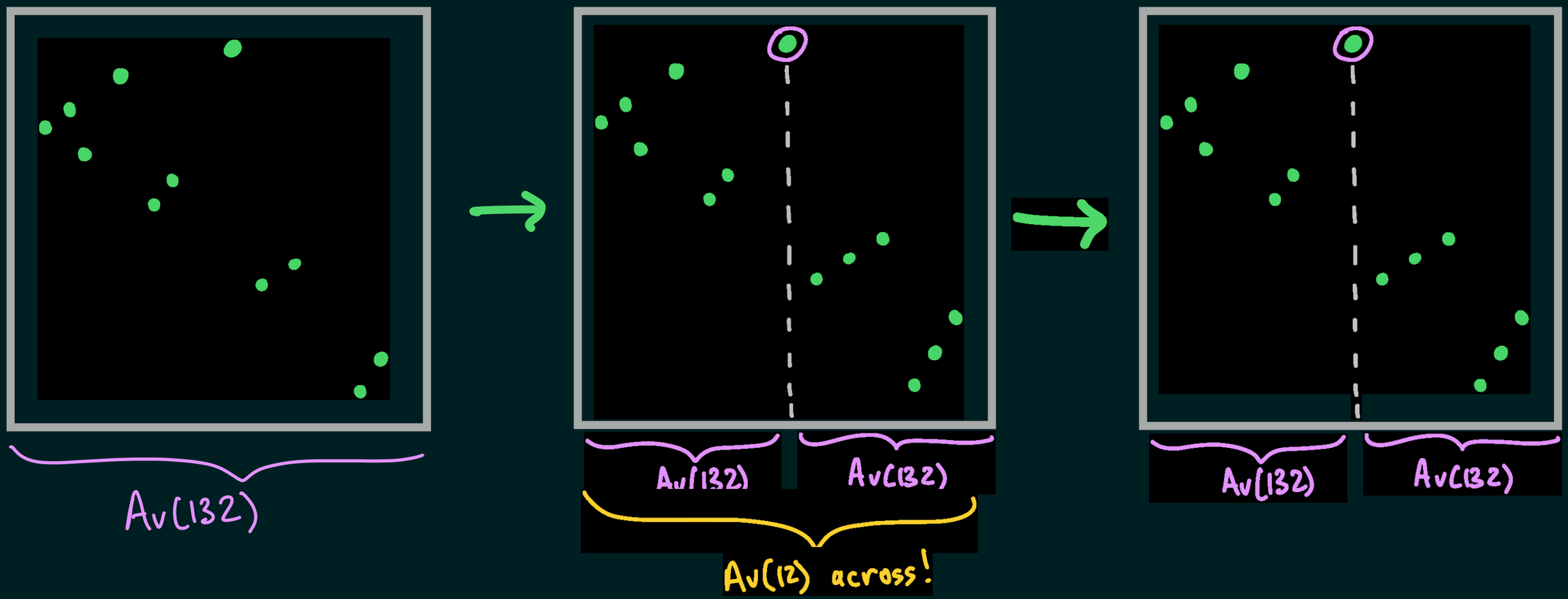
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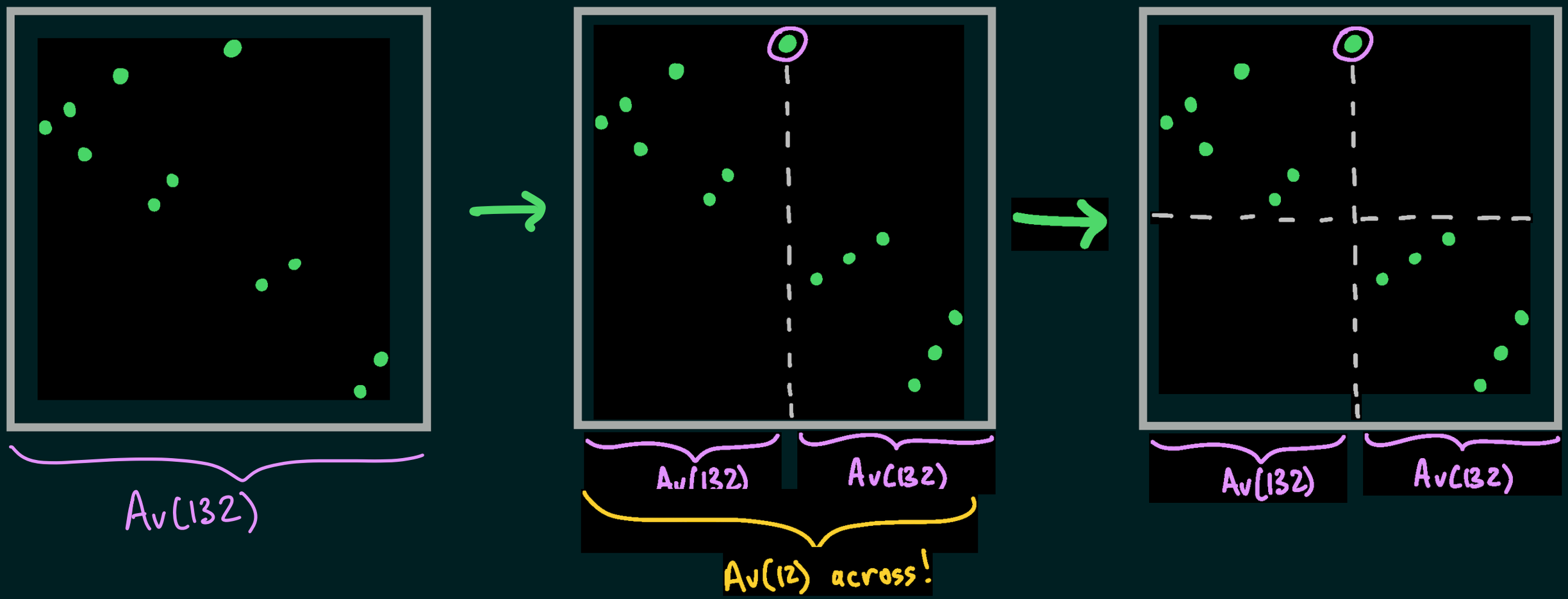

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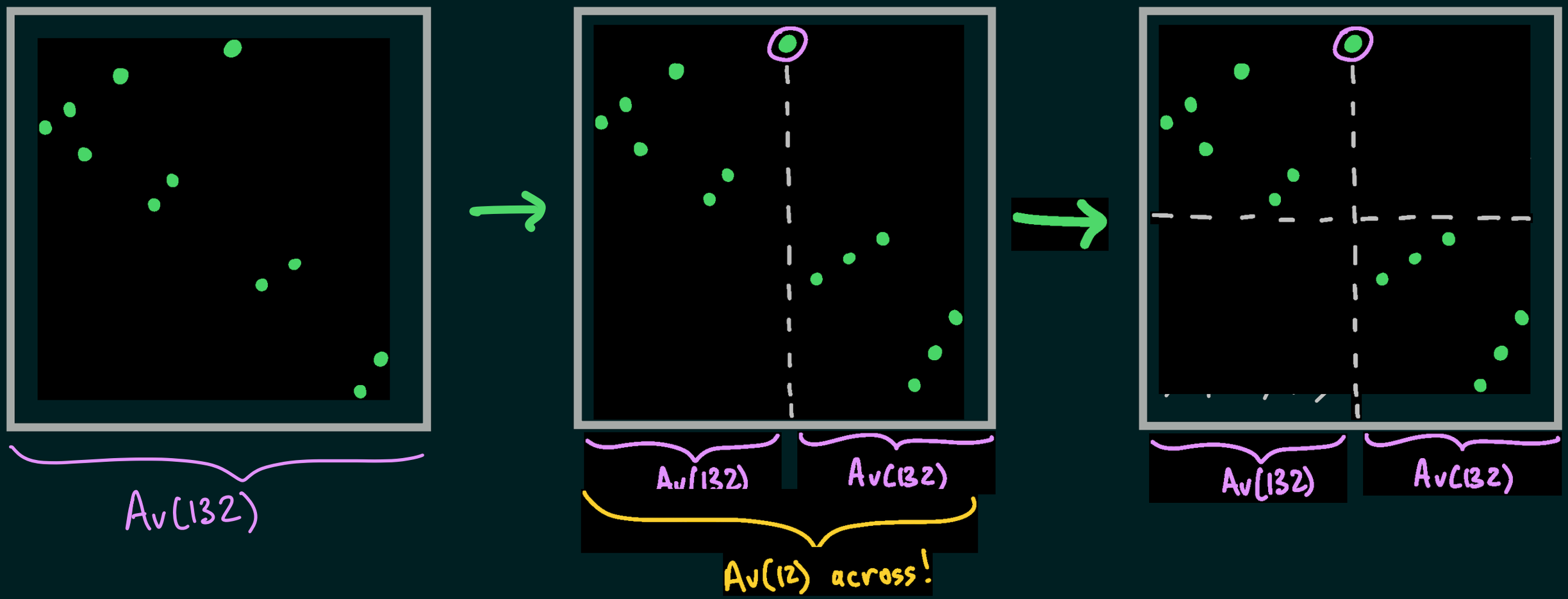

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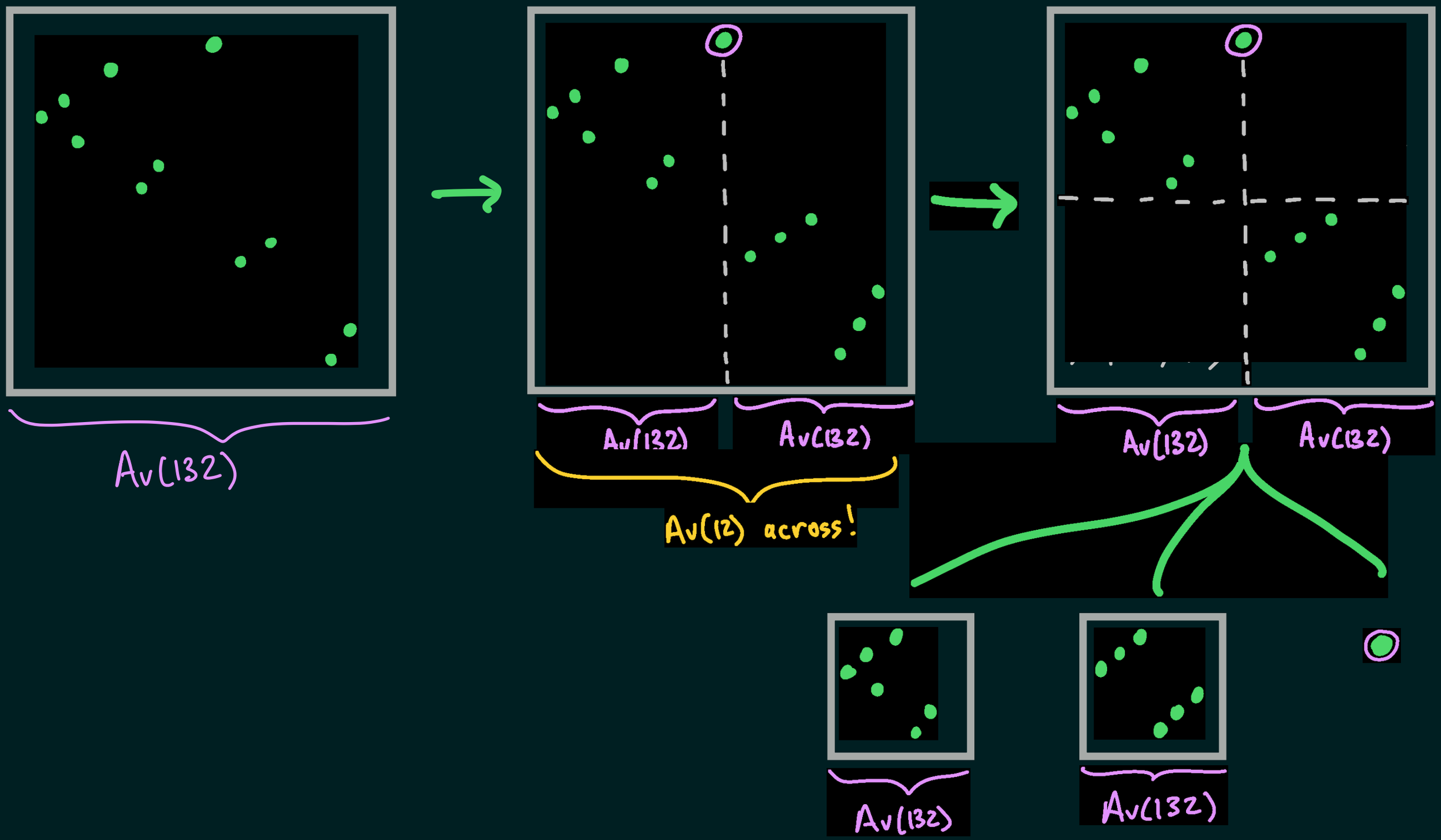

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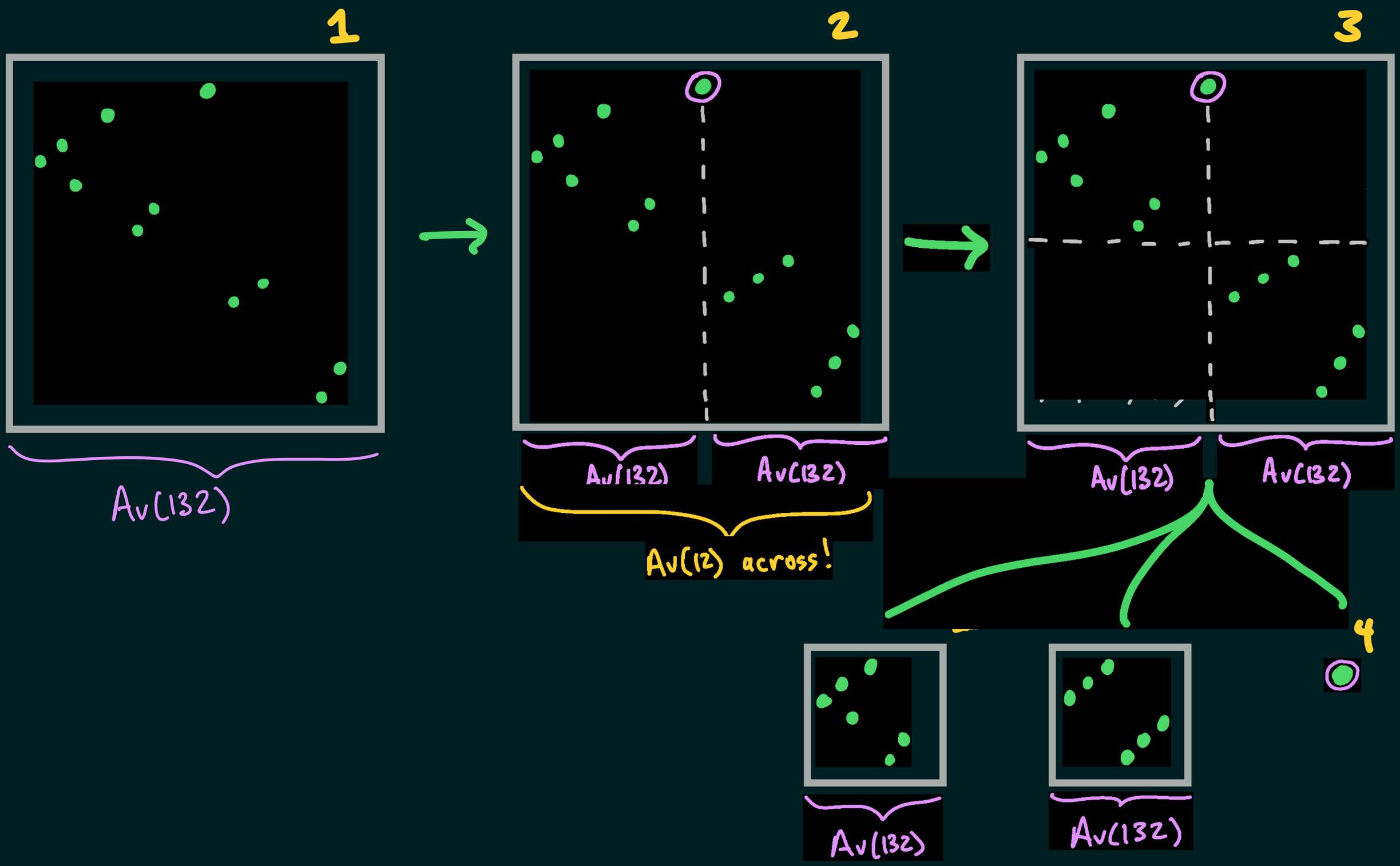

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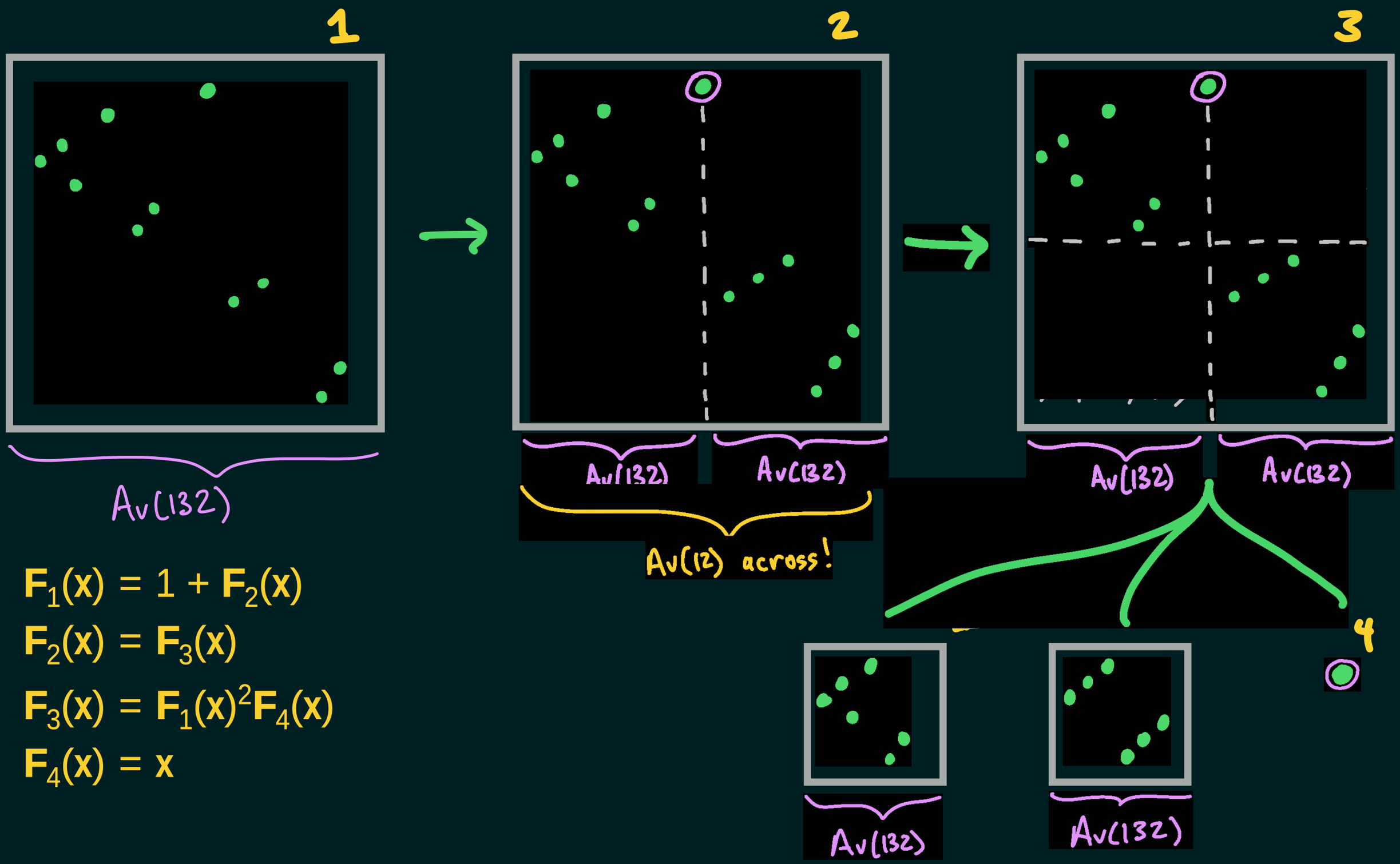

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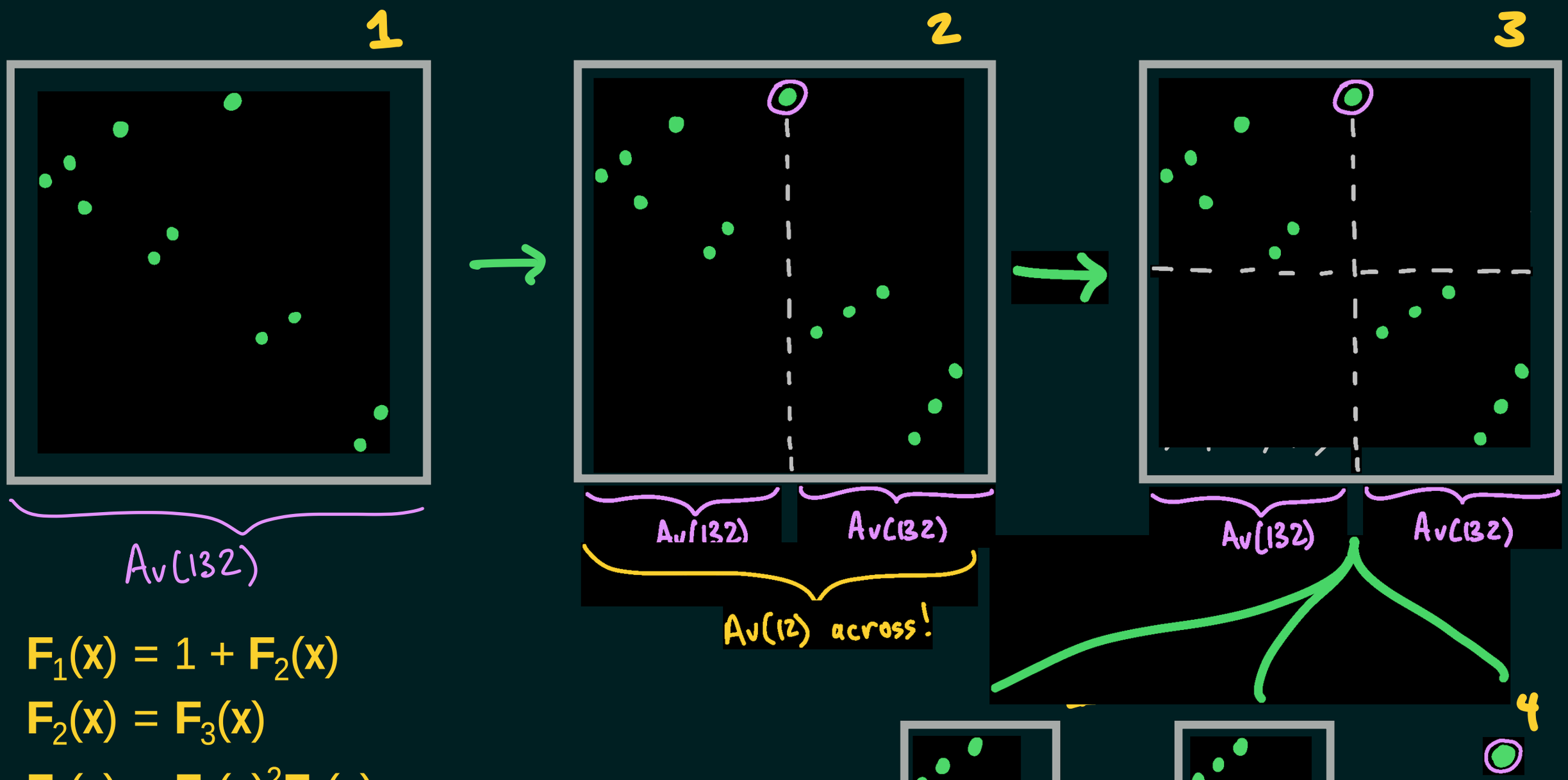

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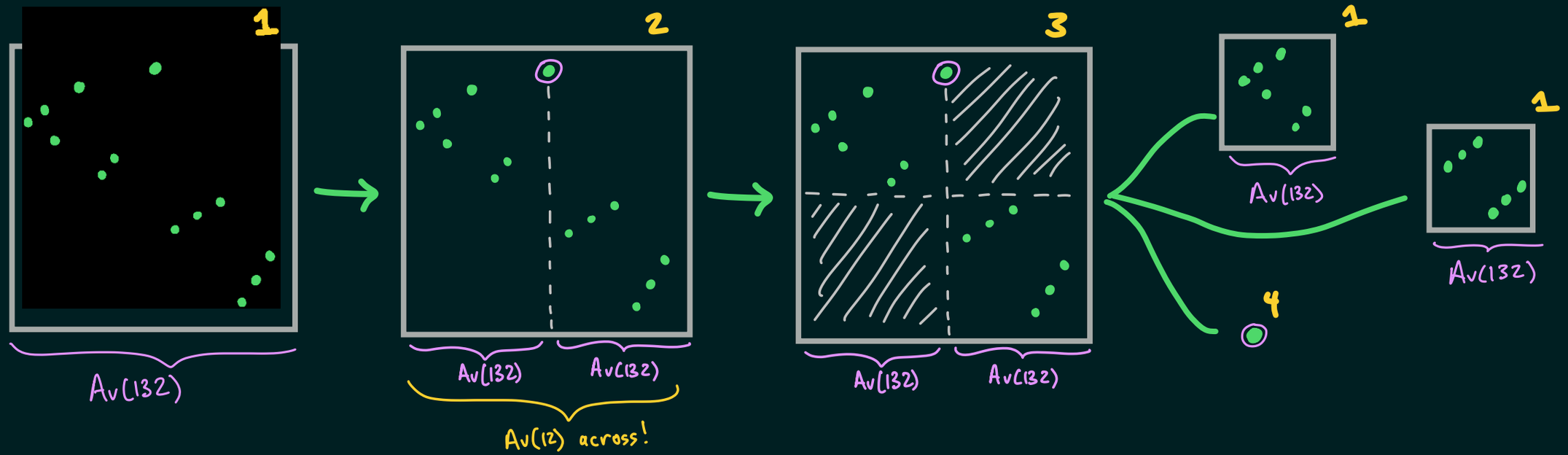


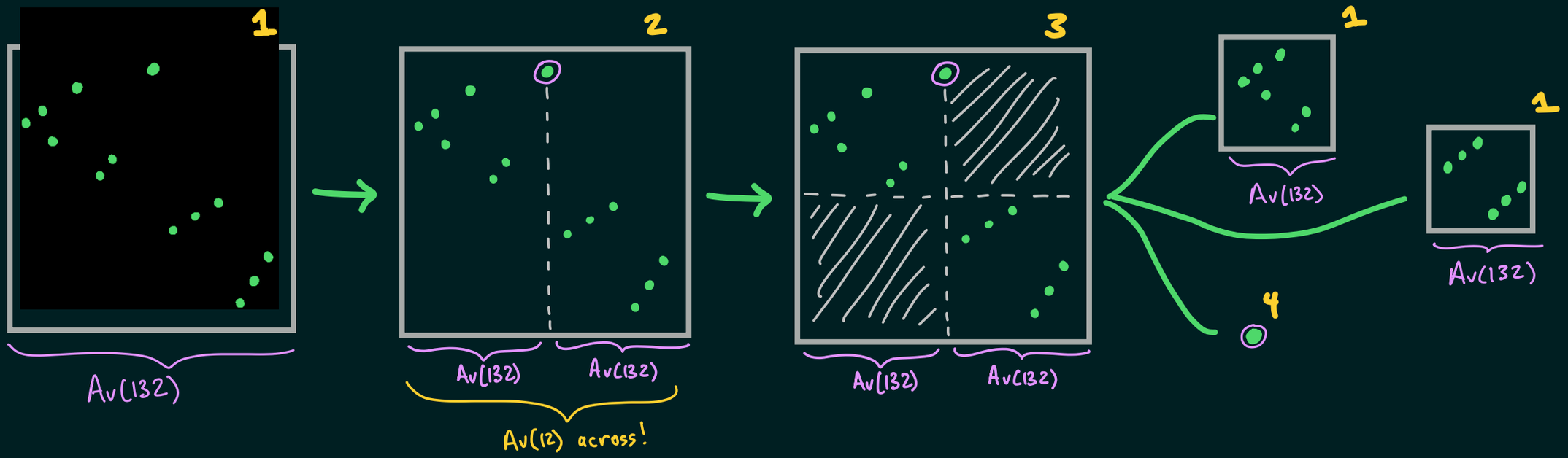
$$F_1(x) = 1 + F_2(x)$$
$$F_2(x) = F_3(x)$$
$$F_3(x) = F_1(x)^2 F_4(x)$$
$$F_4(x) = x$$

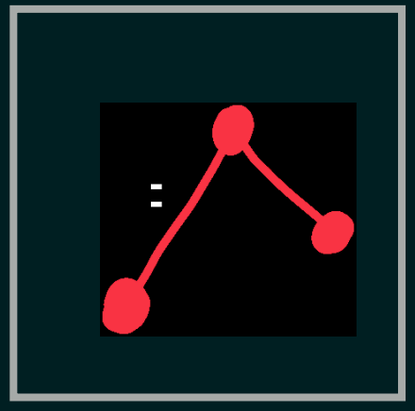


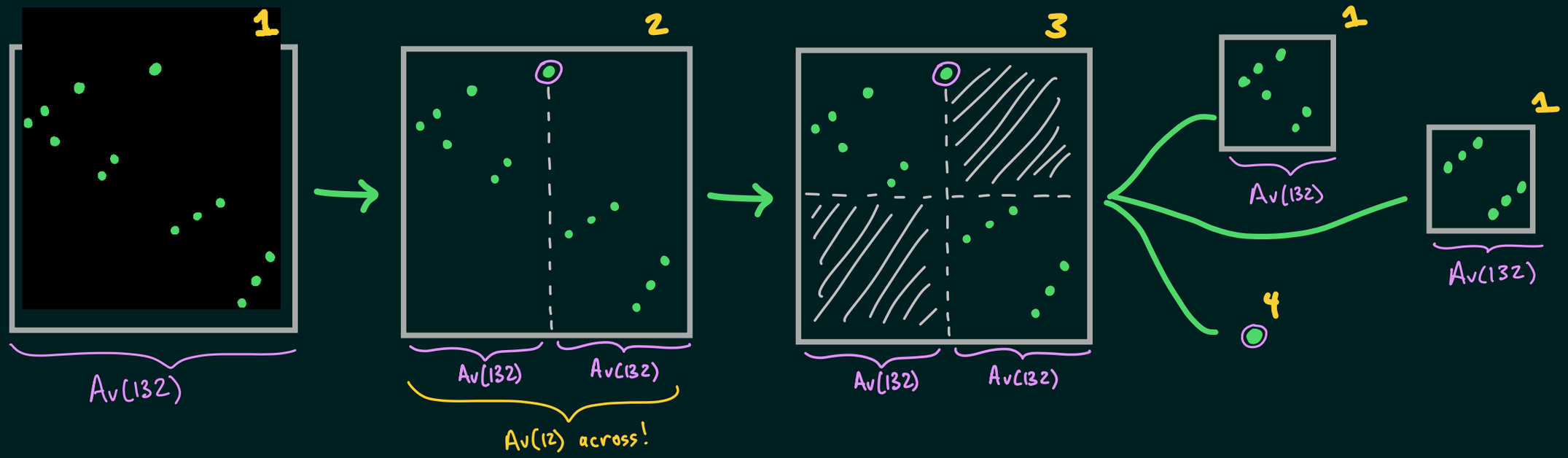
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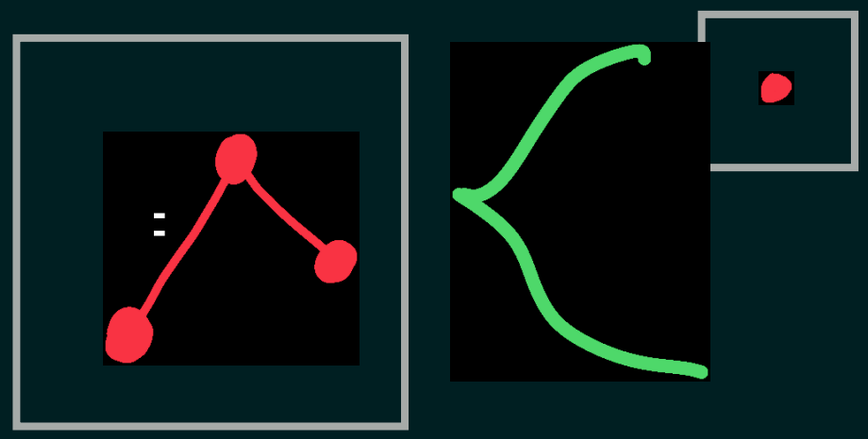
$$F_1(x) = \frac{1 - \sqrt{1 - 4x}}{2x}$$

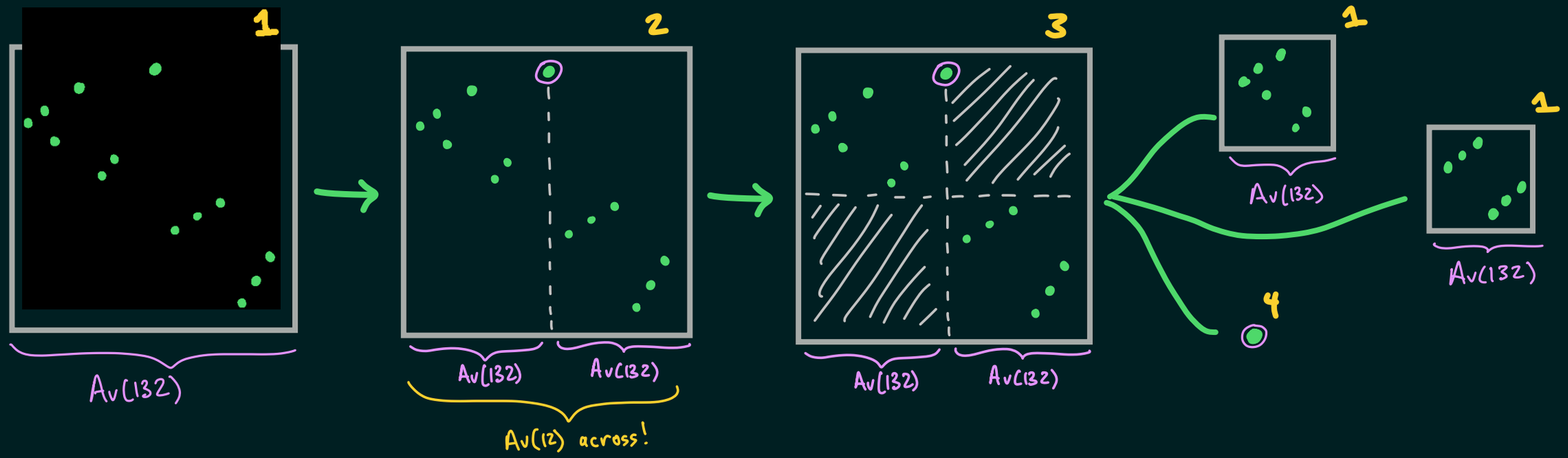

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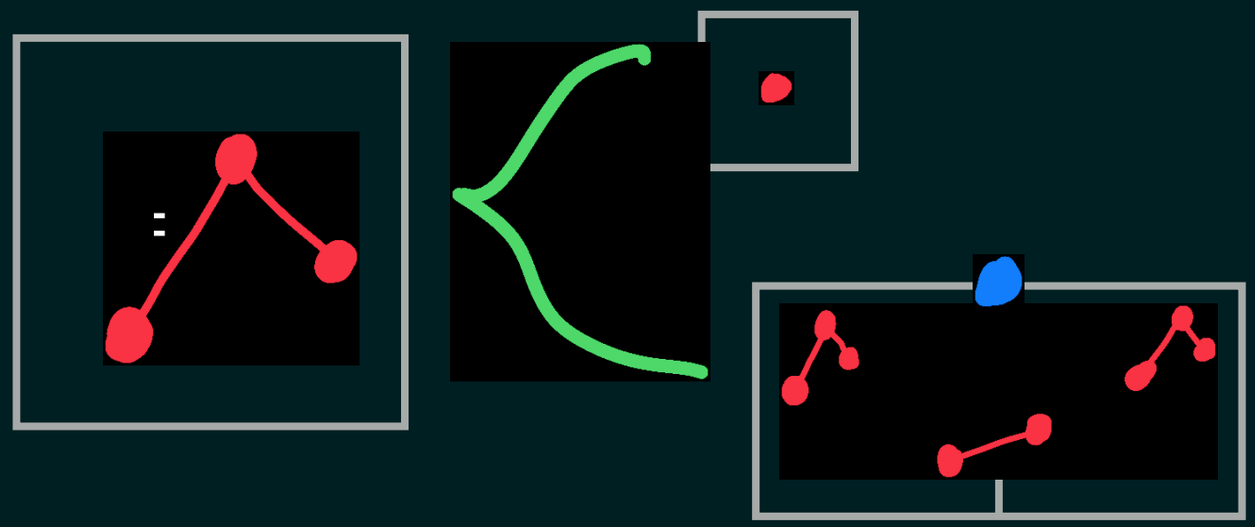

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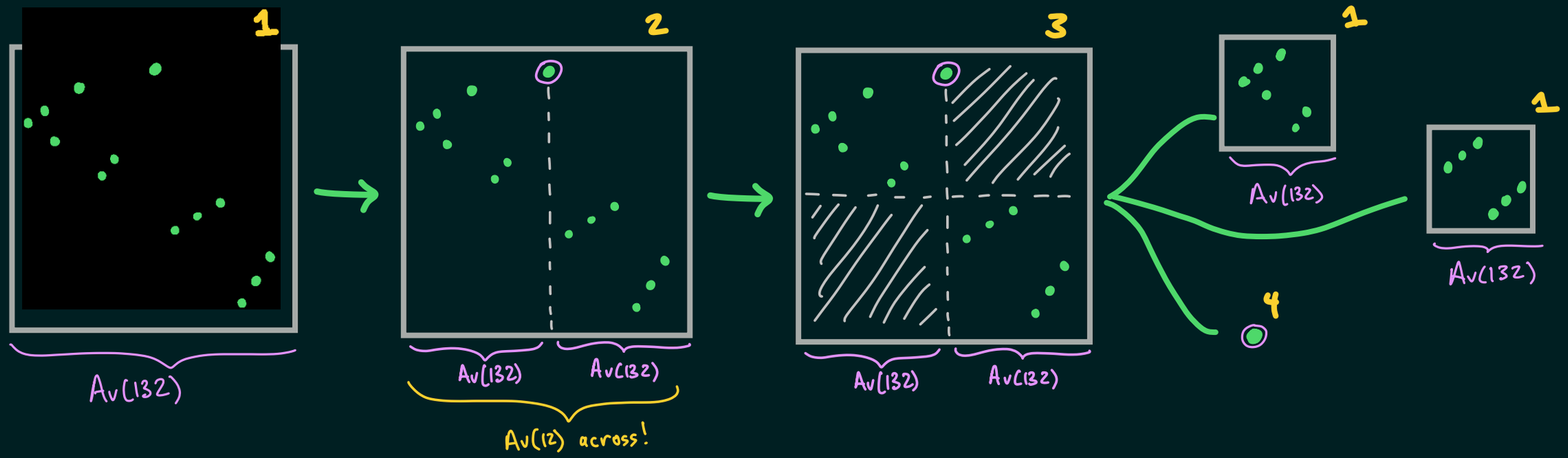


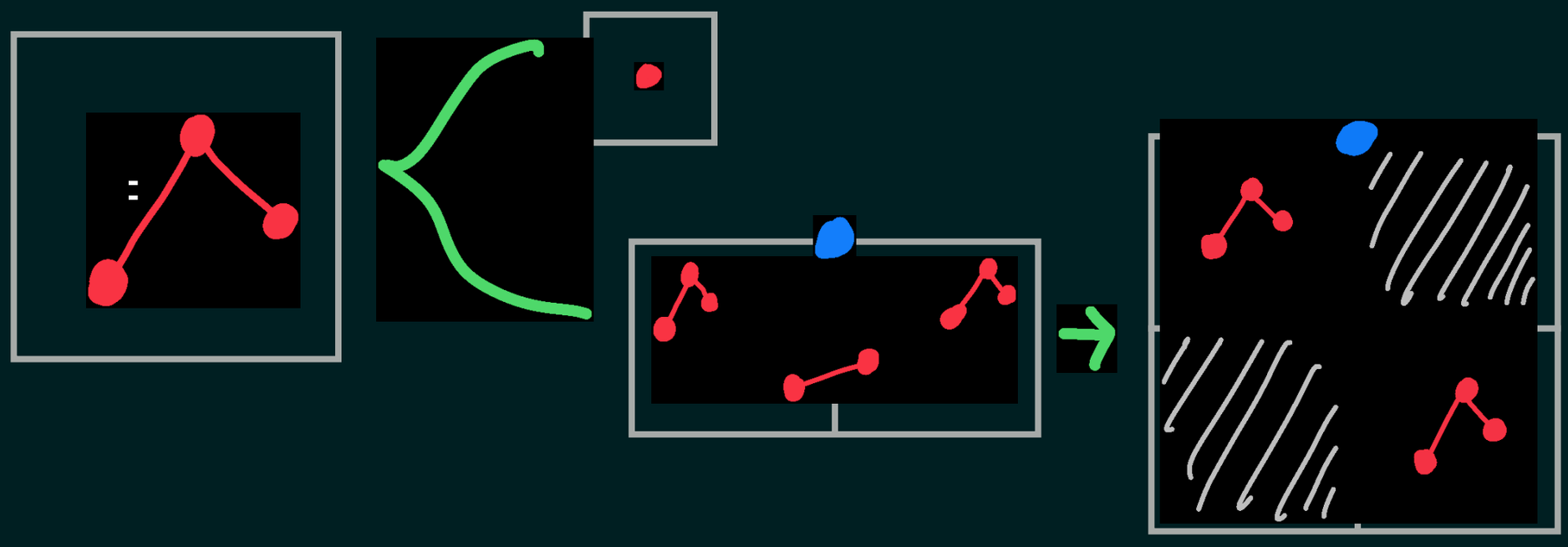

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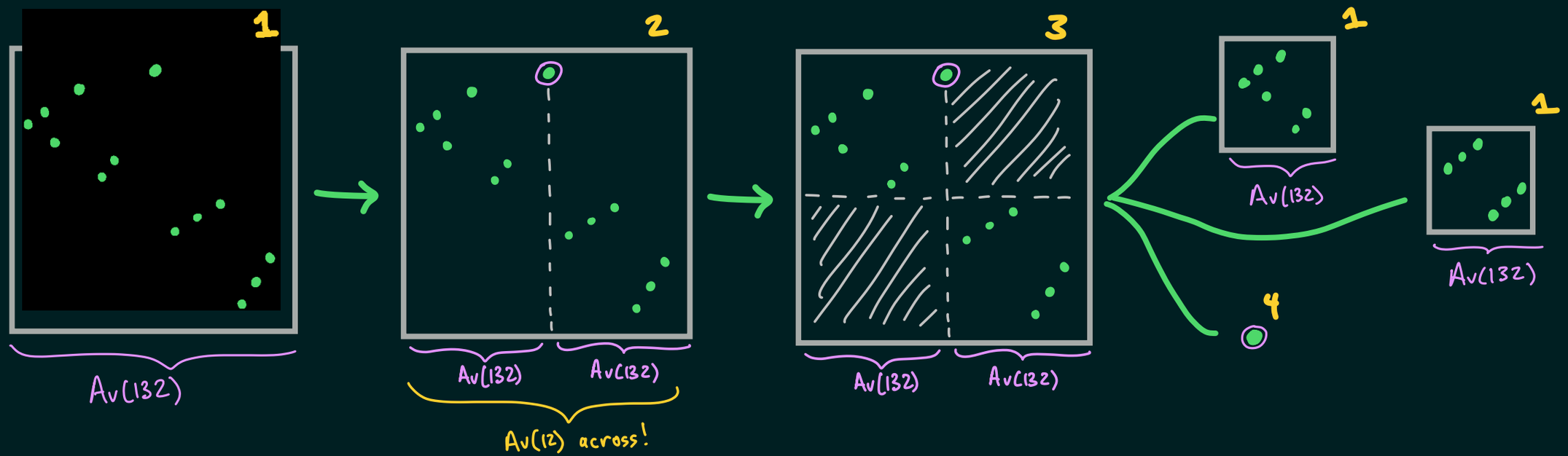


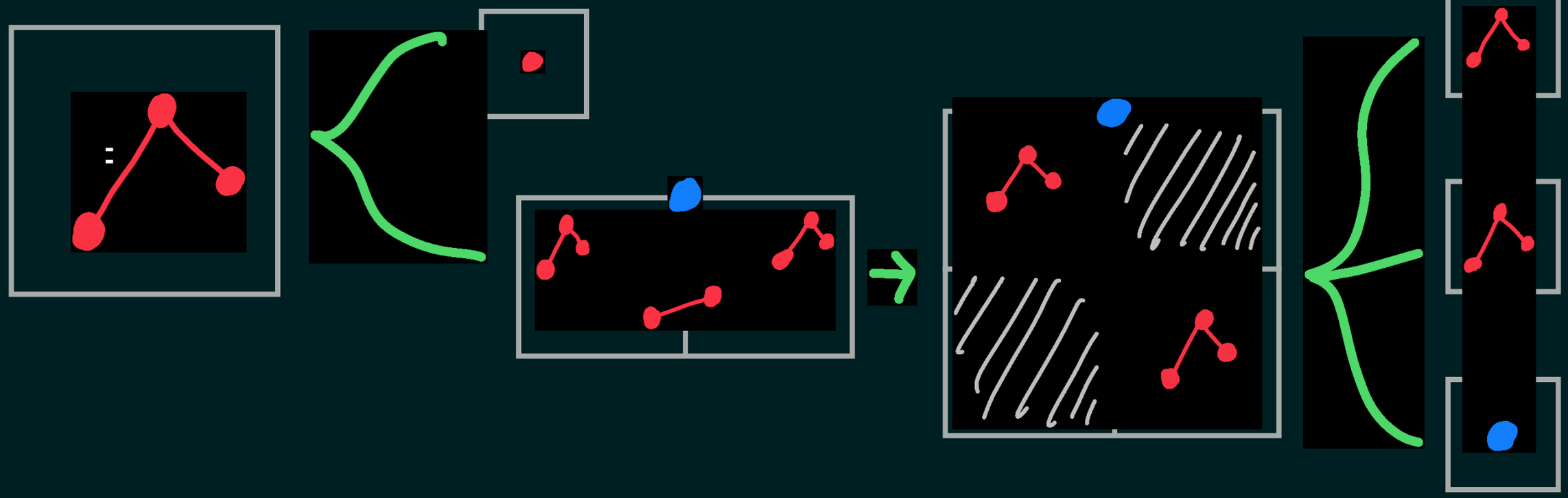

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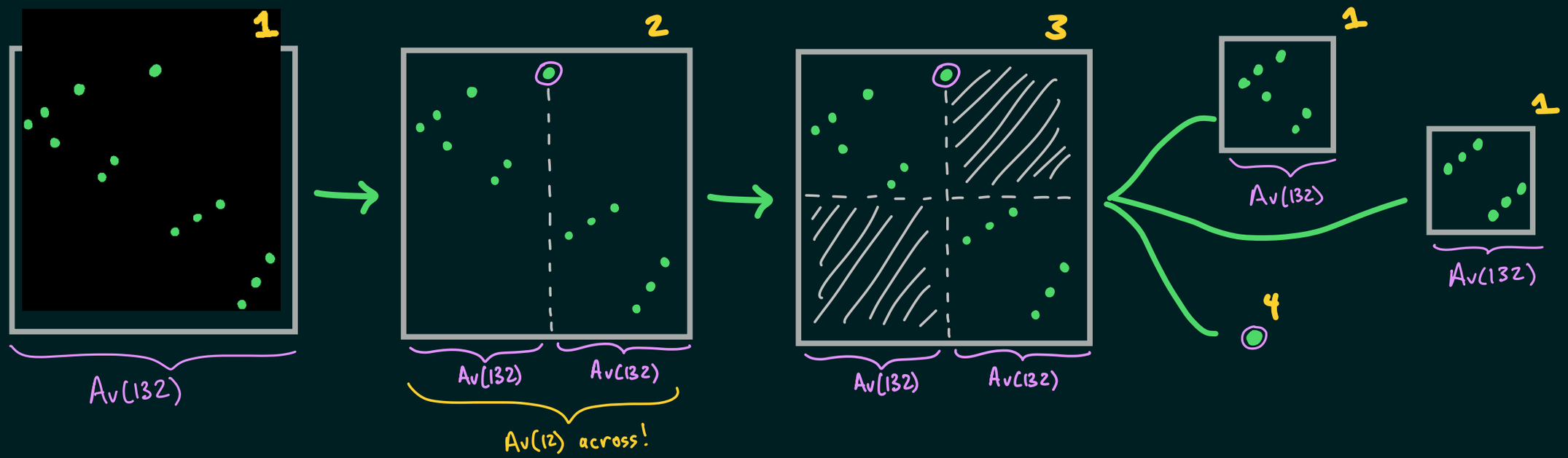


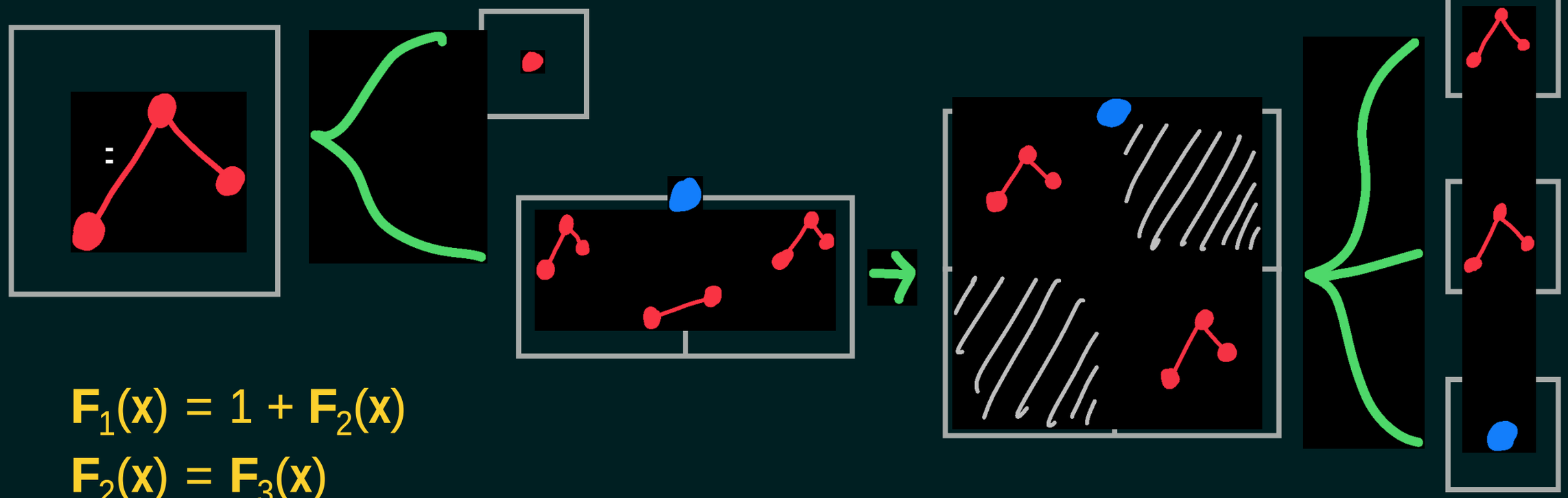

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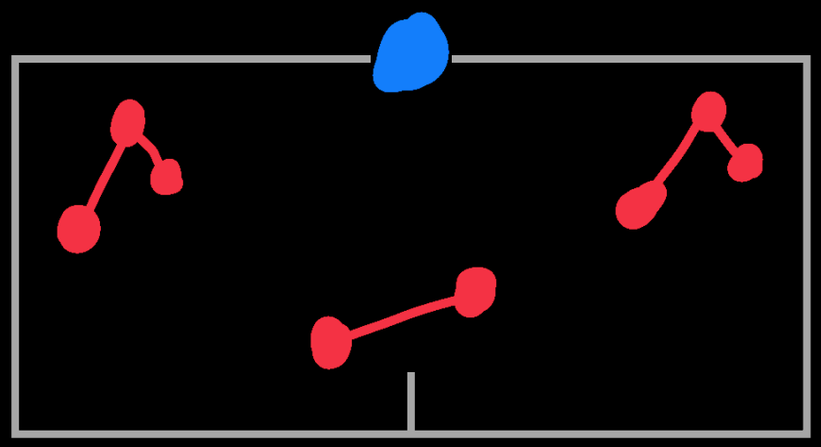
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$$F_4(x) = x$$

$$F_1(x) = \frac{1 - \sqrt{1 - 4x}}{2x}$$

We call this a tiling .

It represents the set of permutations (really, a set of gridded permutations) that you can draw on top of the picture such that you

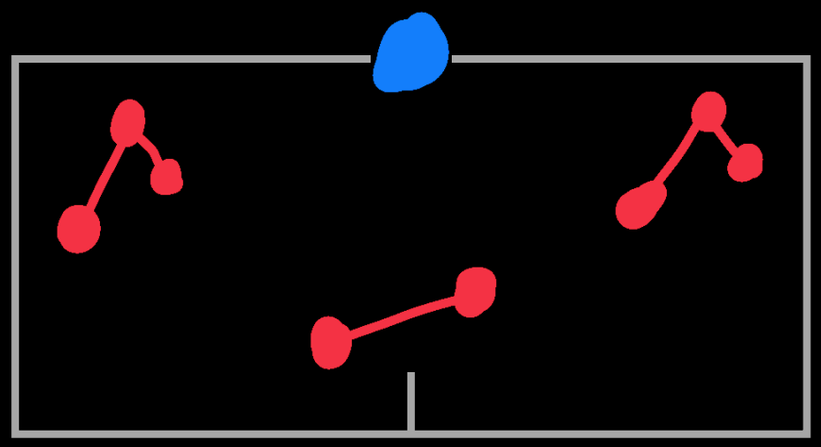
- Don’t make any of the red things (“obstructions”)
- Do make every blue thing (“requirements”)

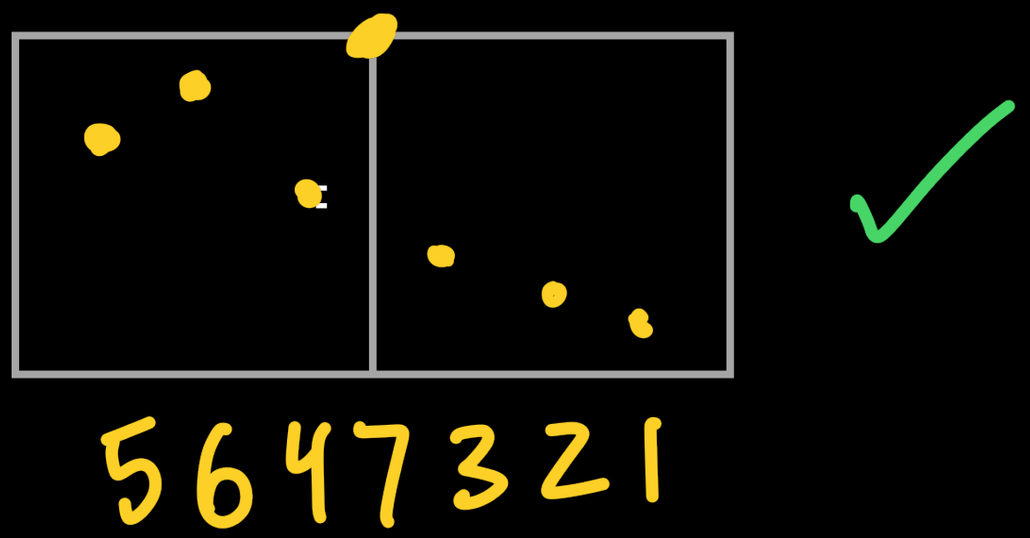


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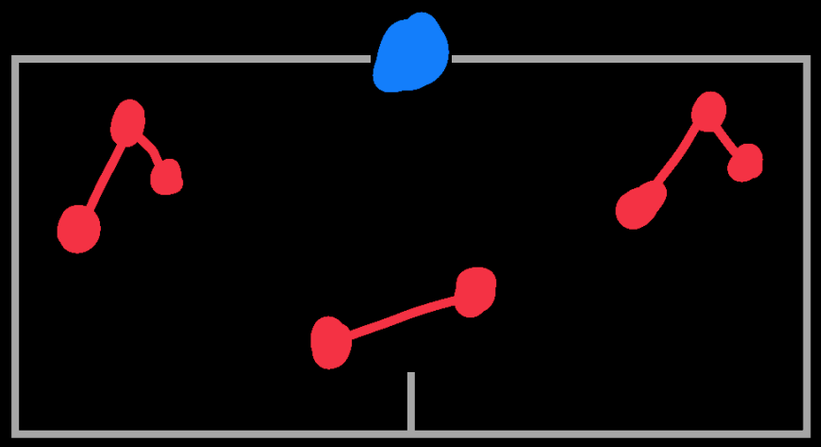


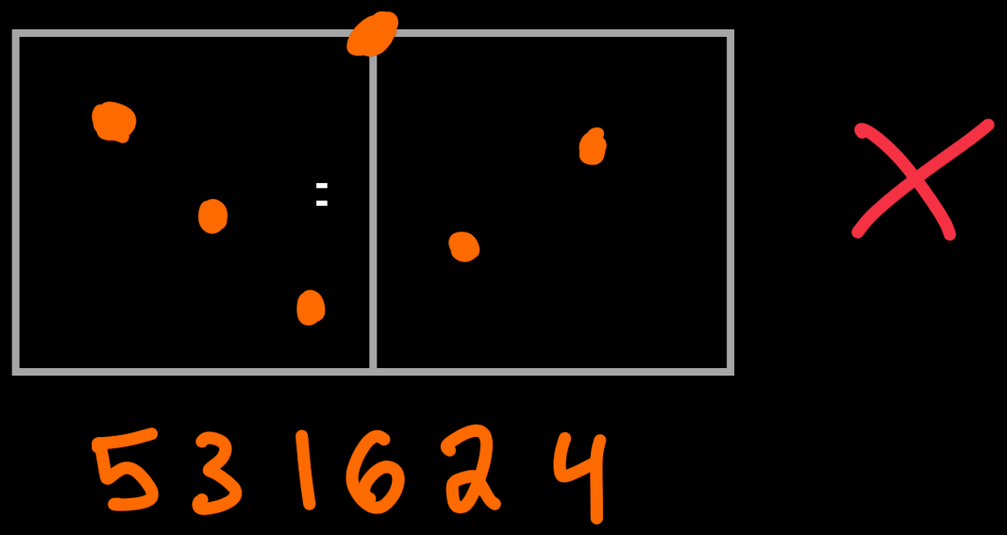


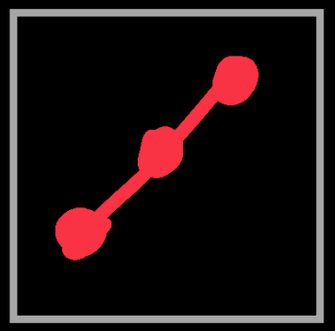
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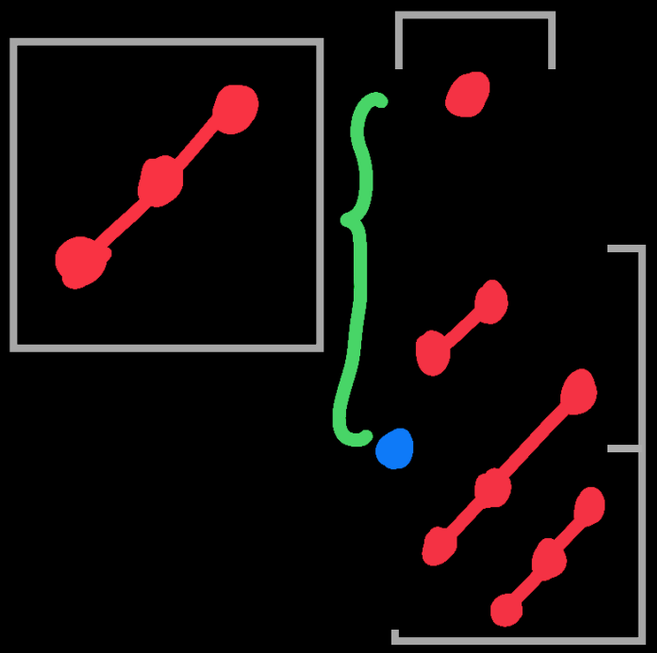
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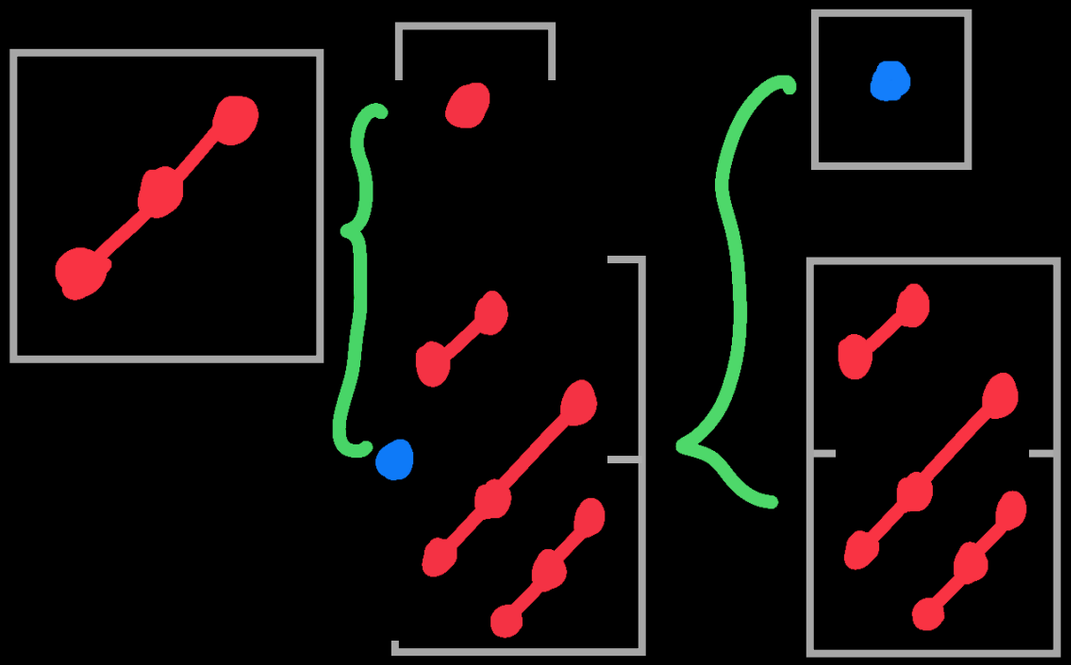
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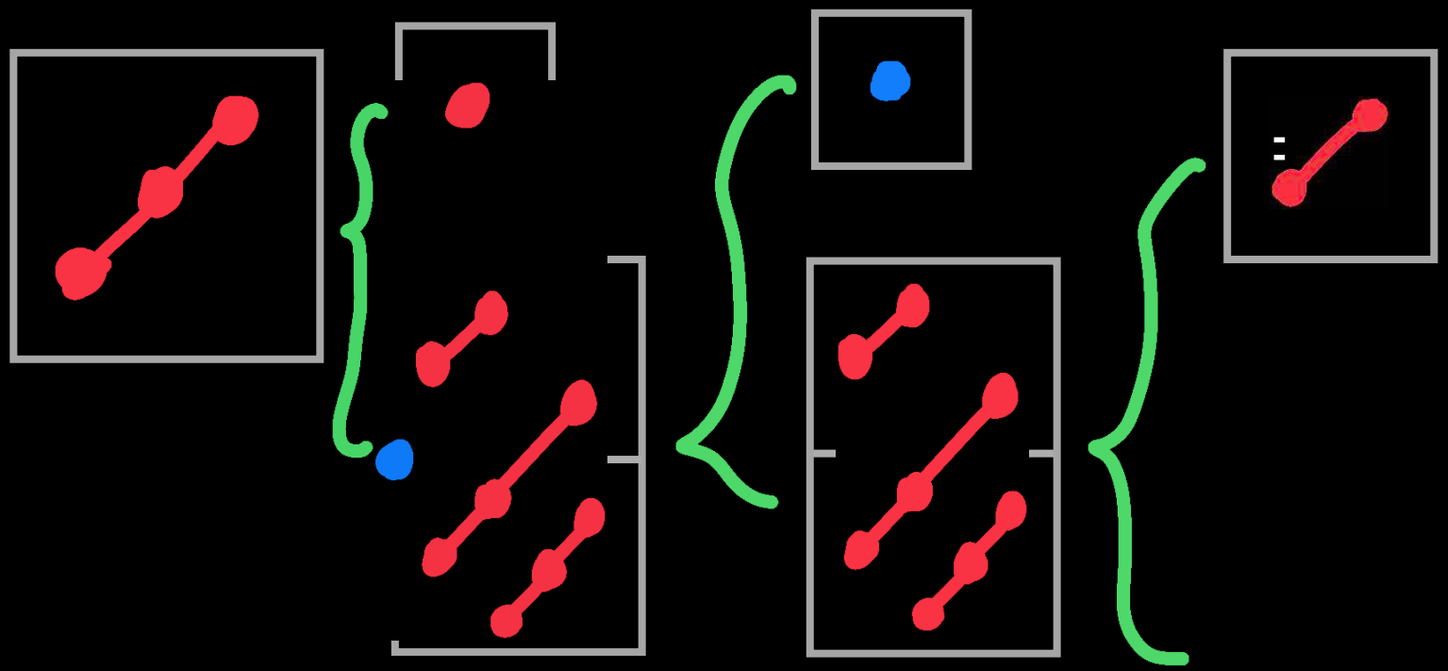


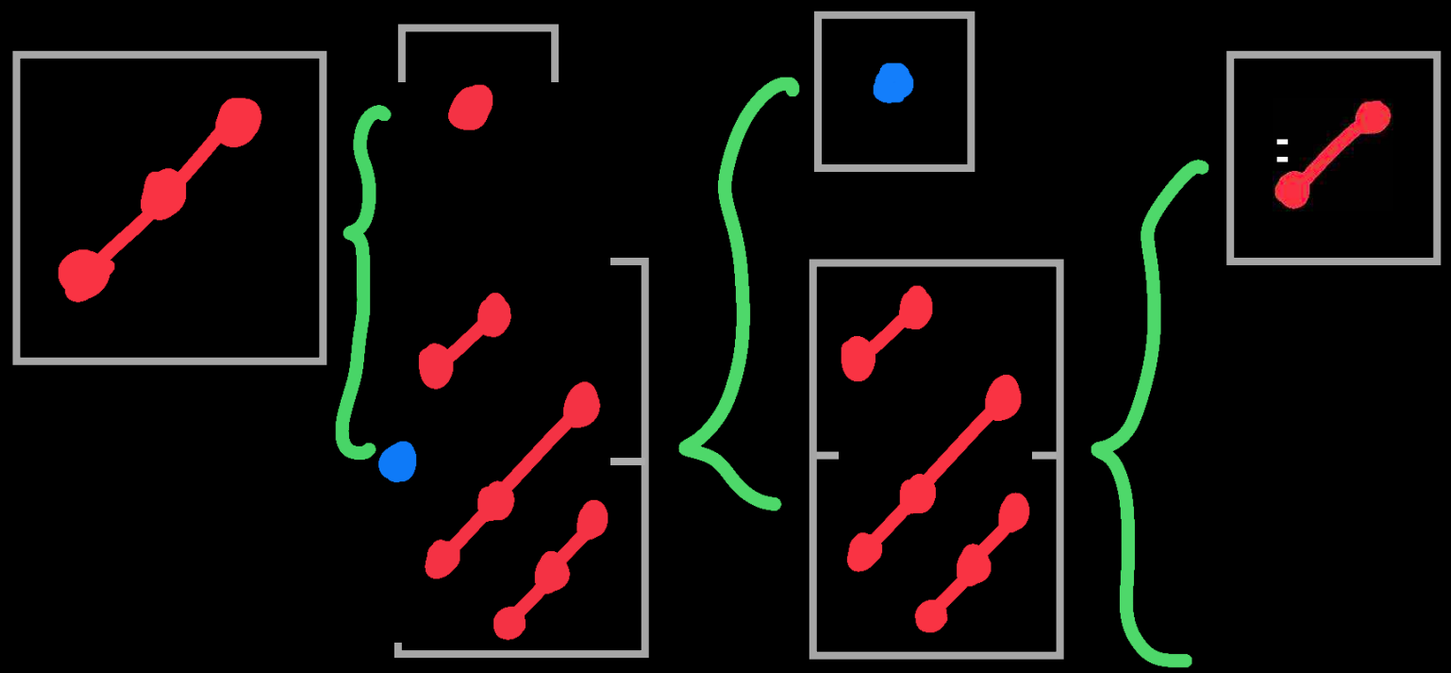


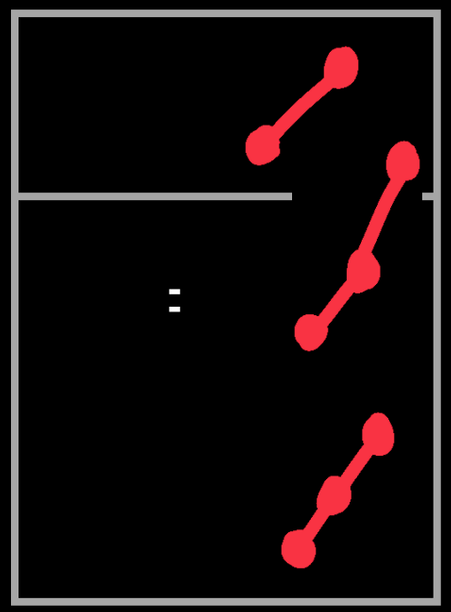


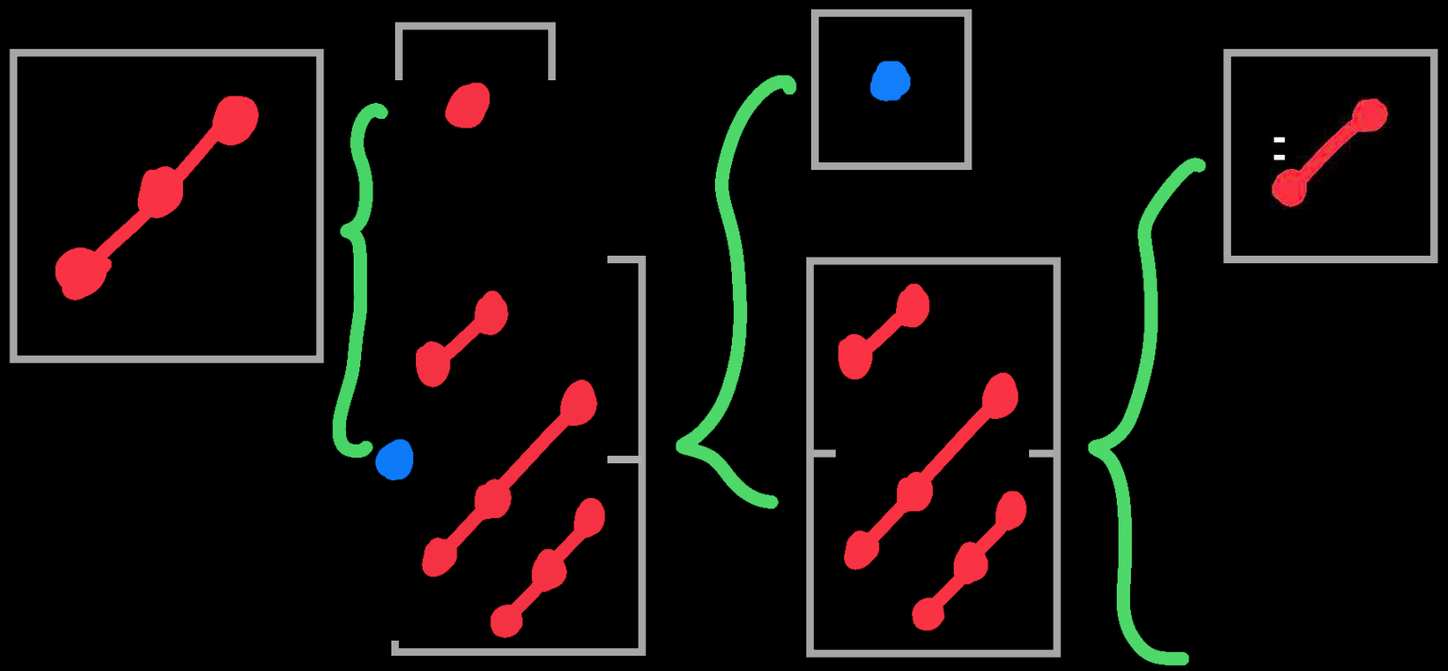


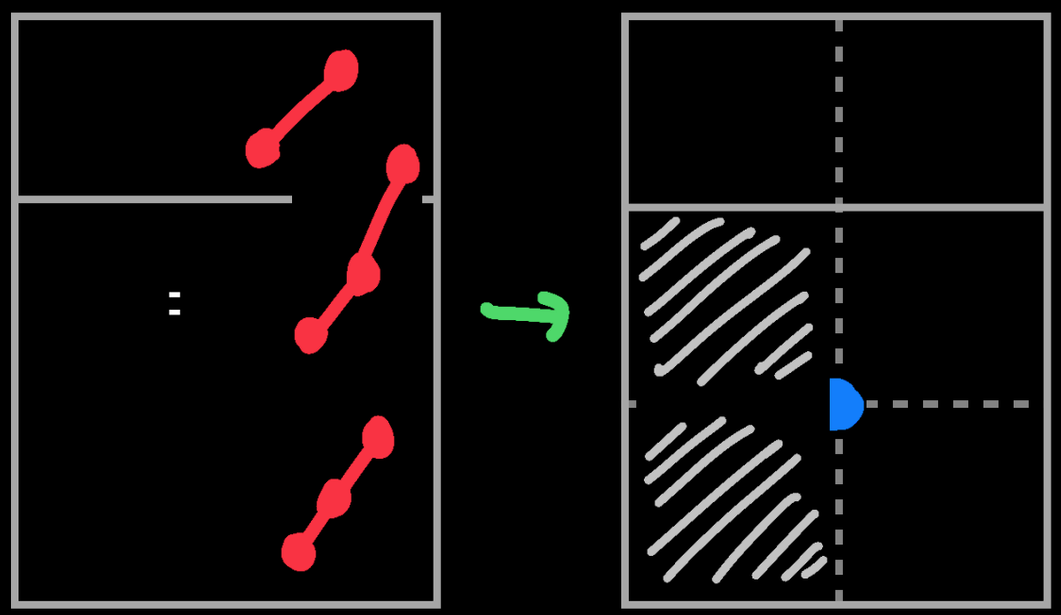


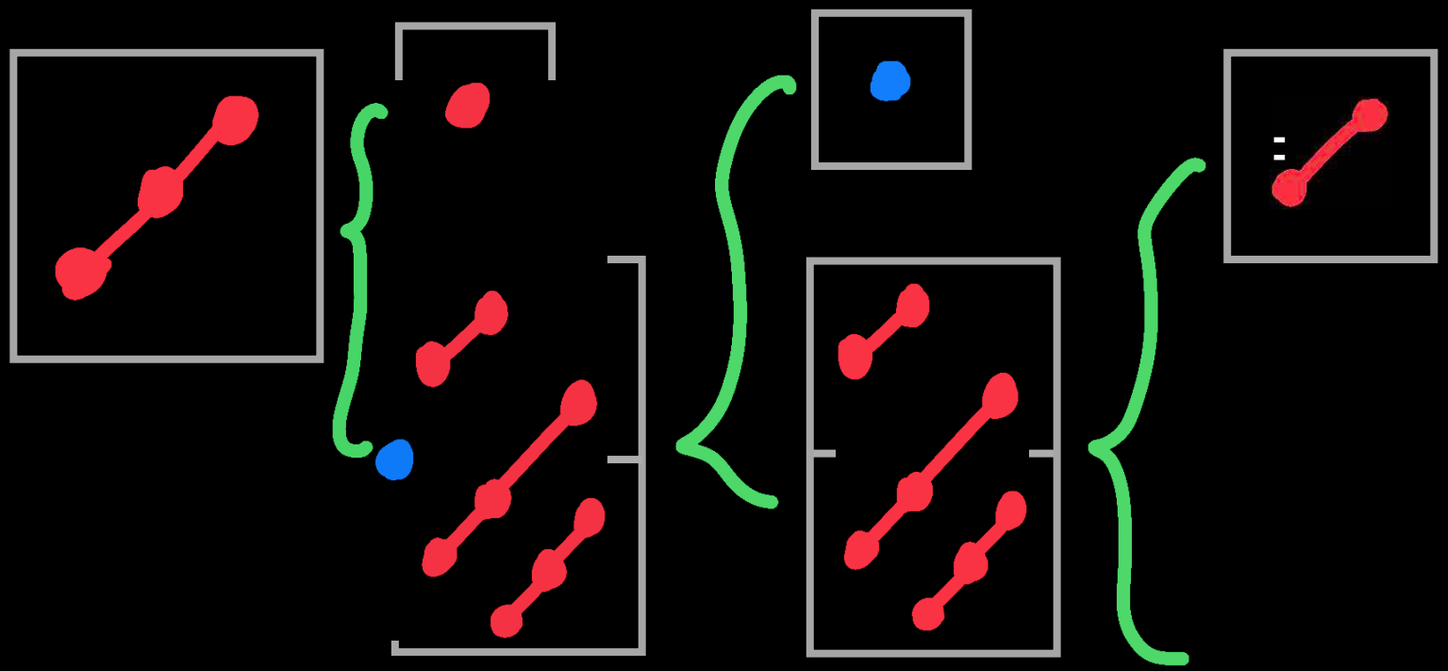


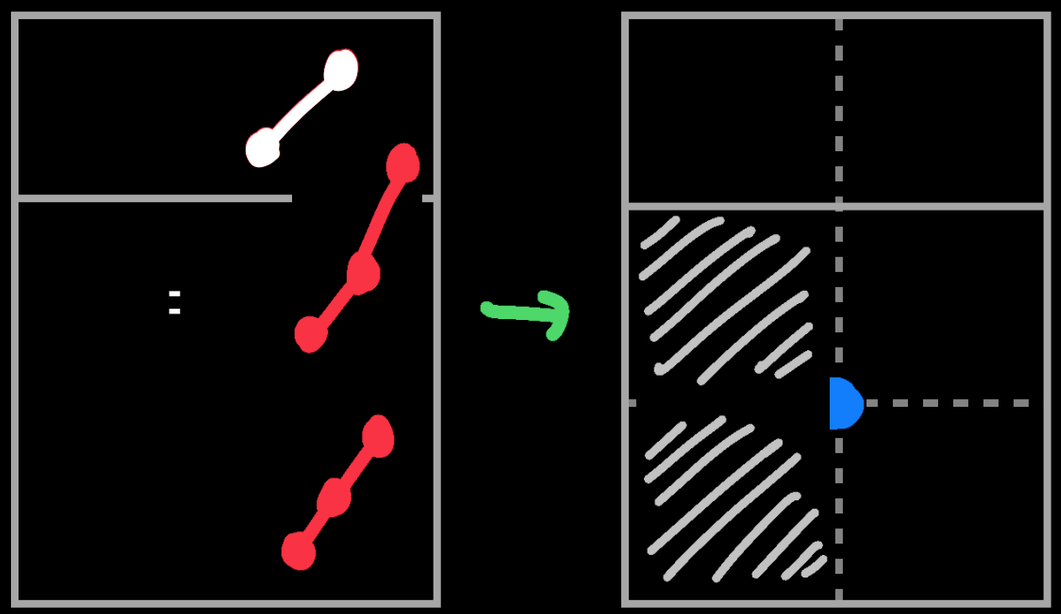


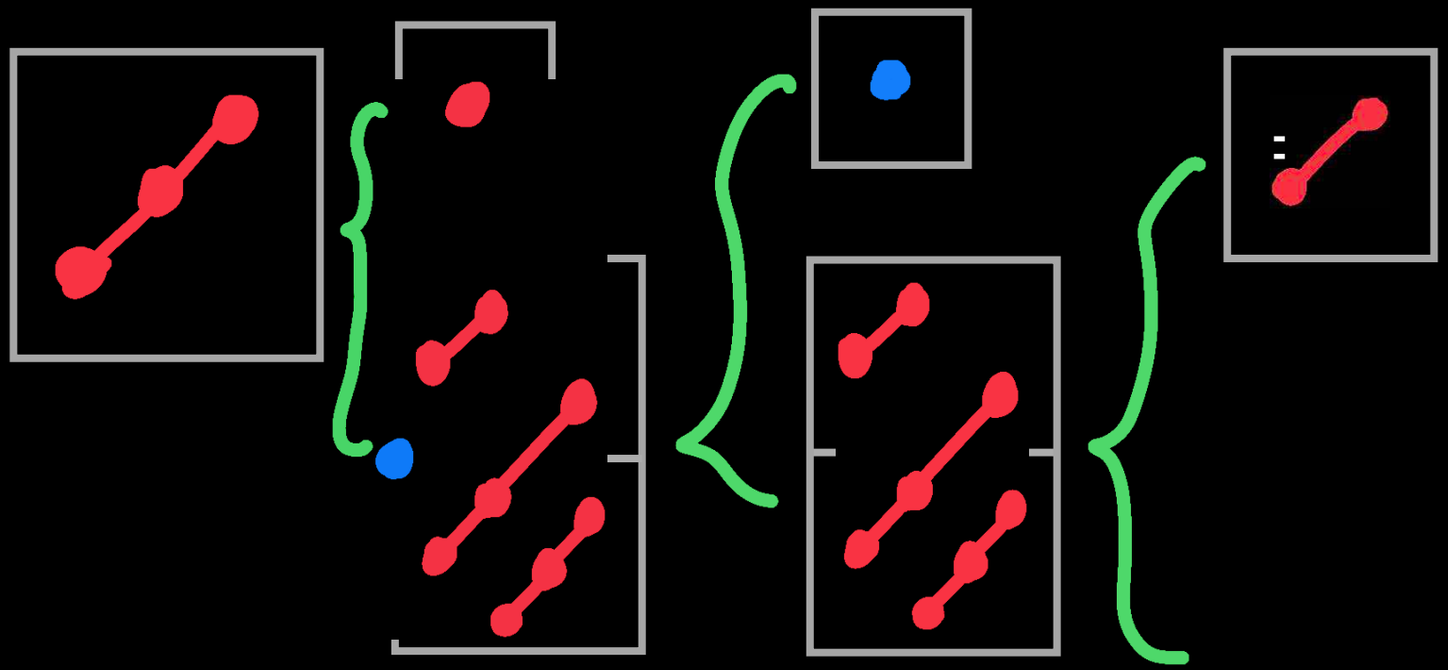


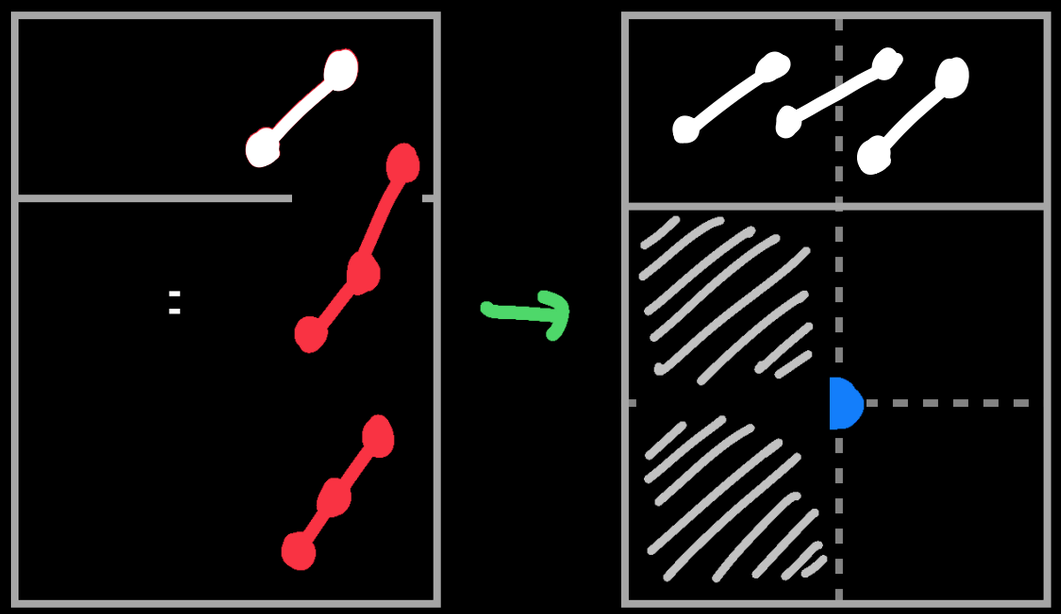


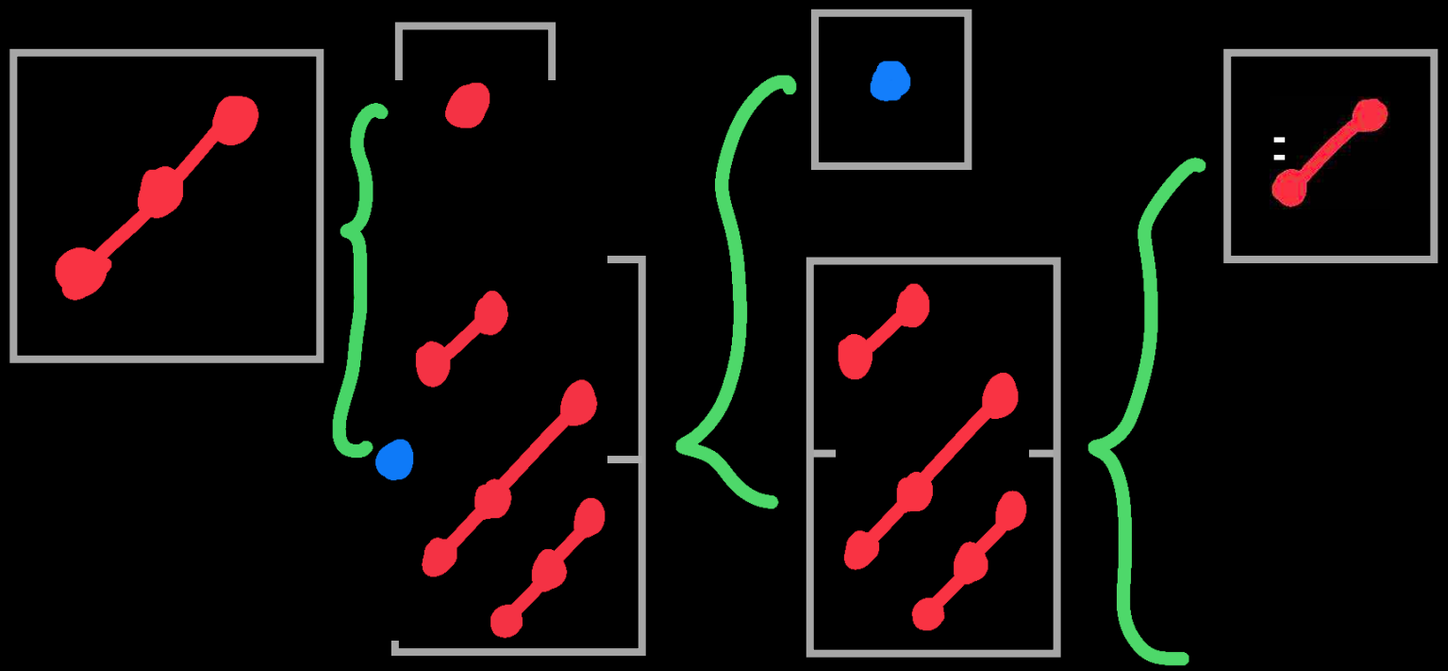


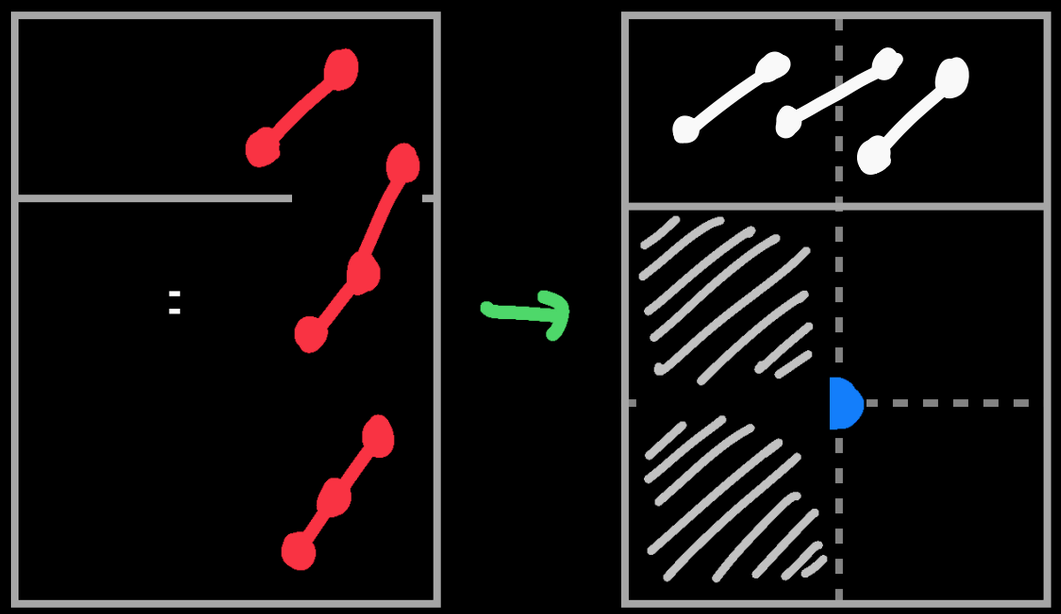


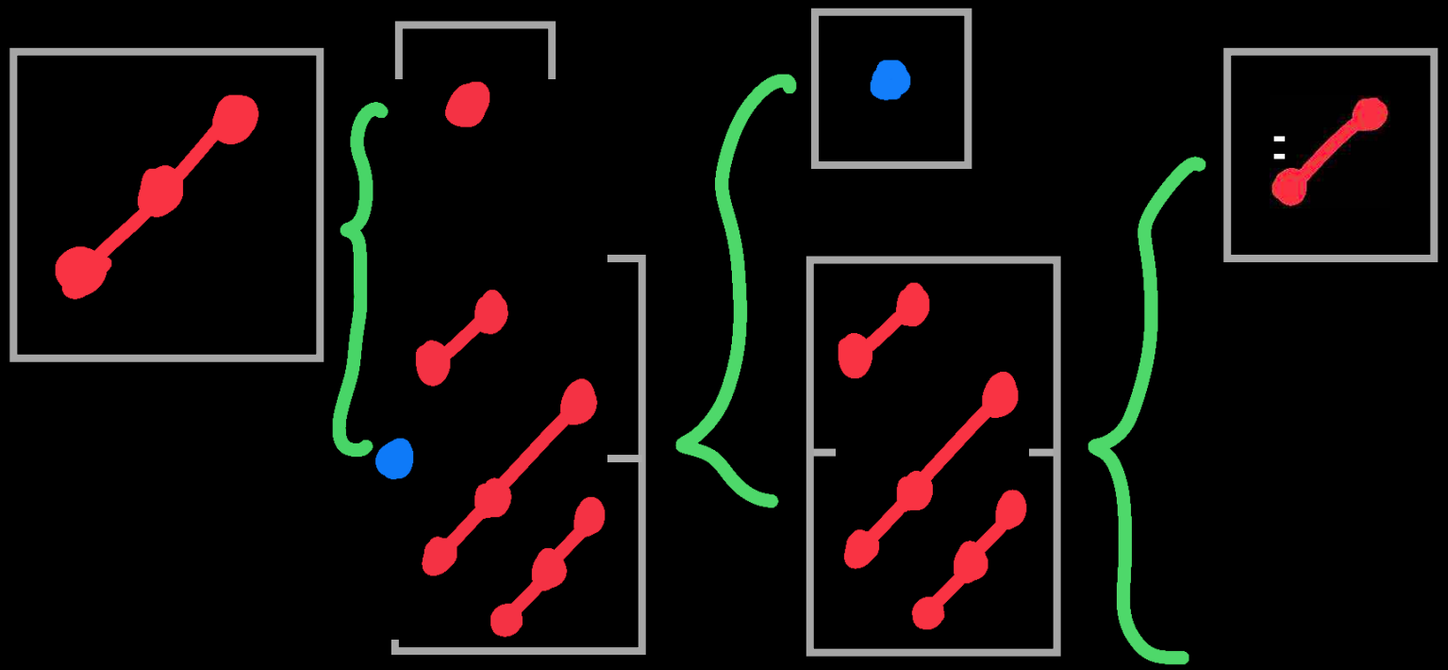


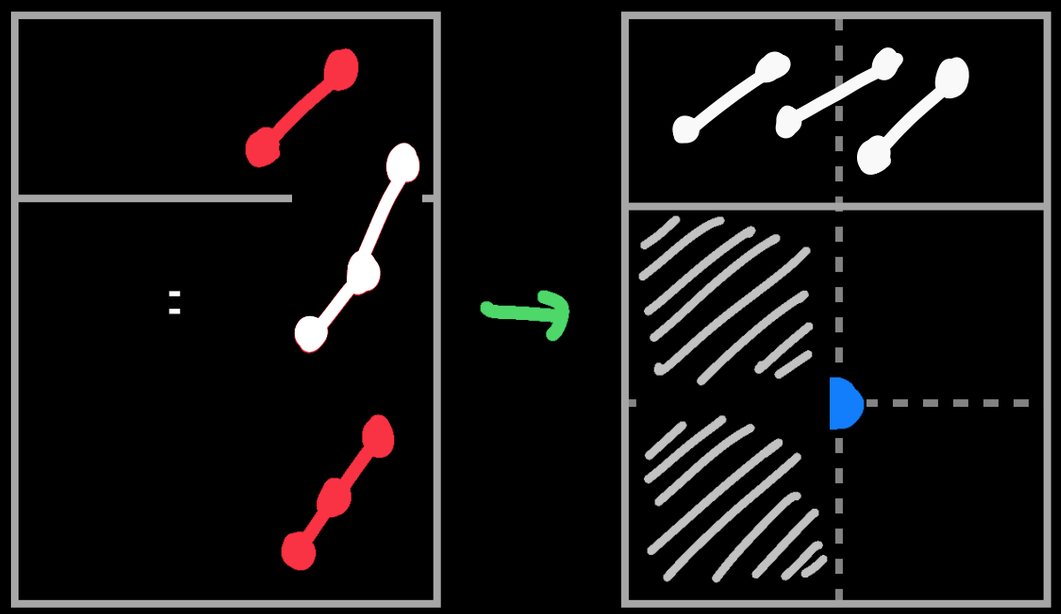


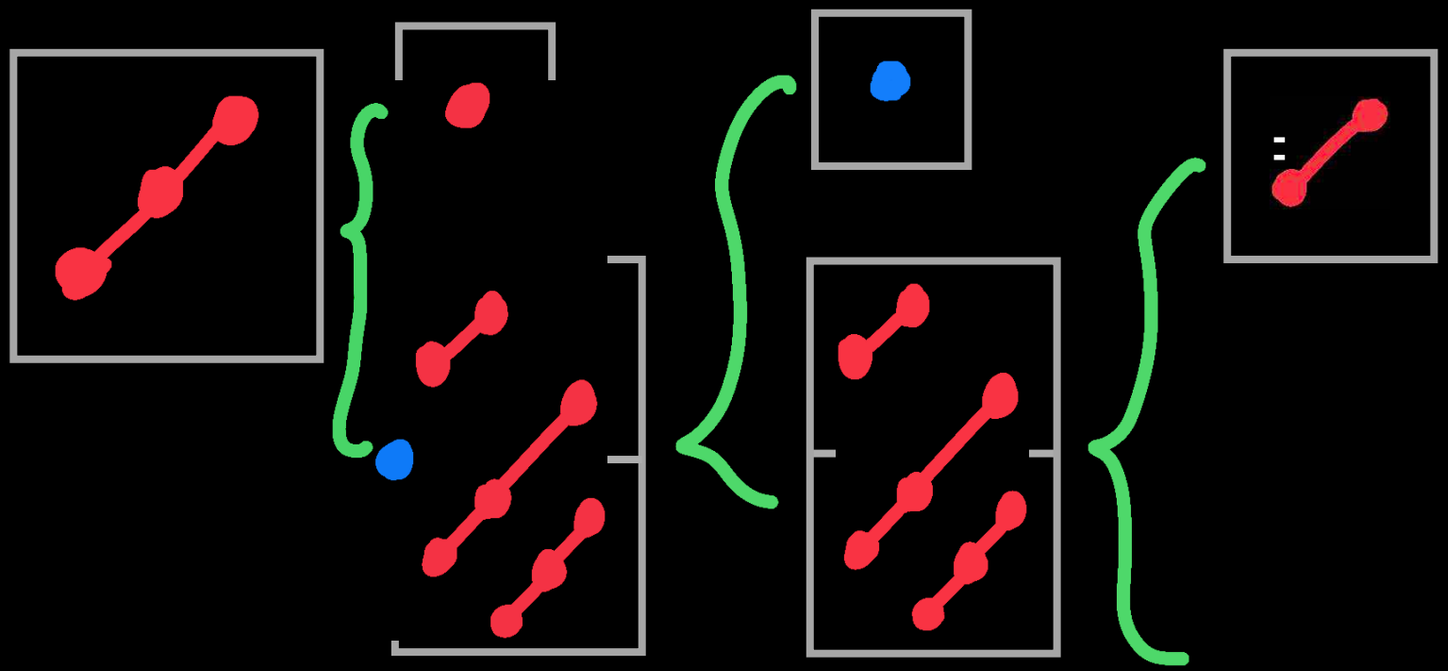


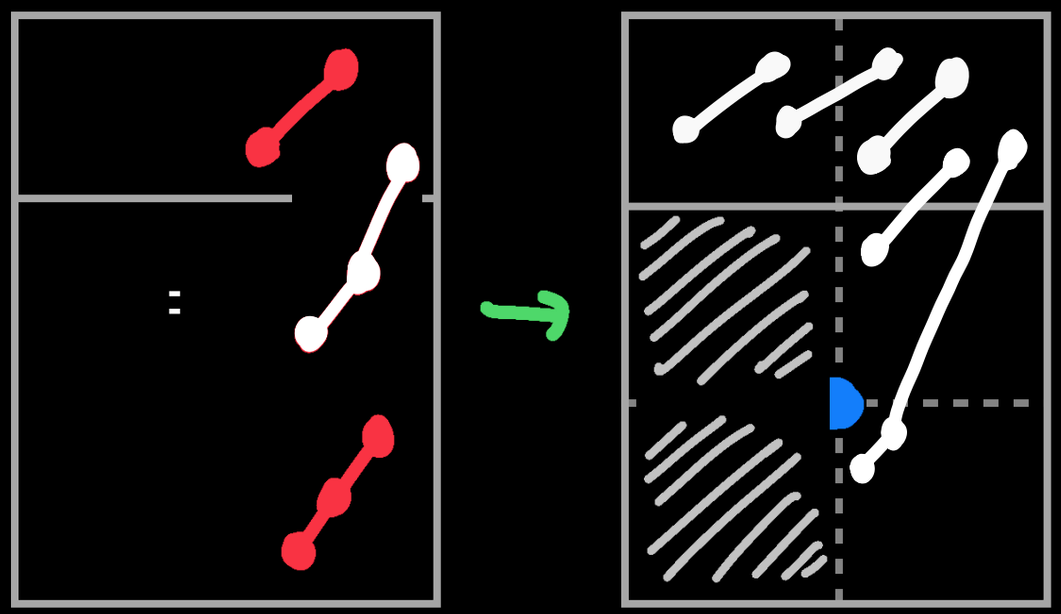


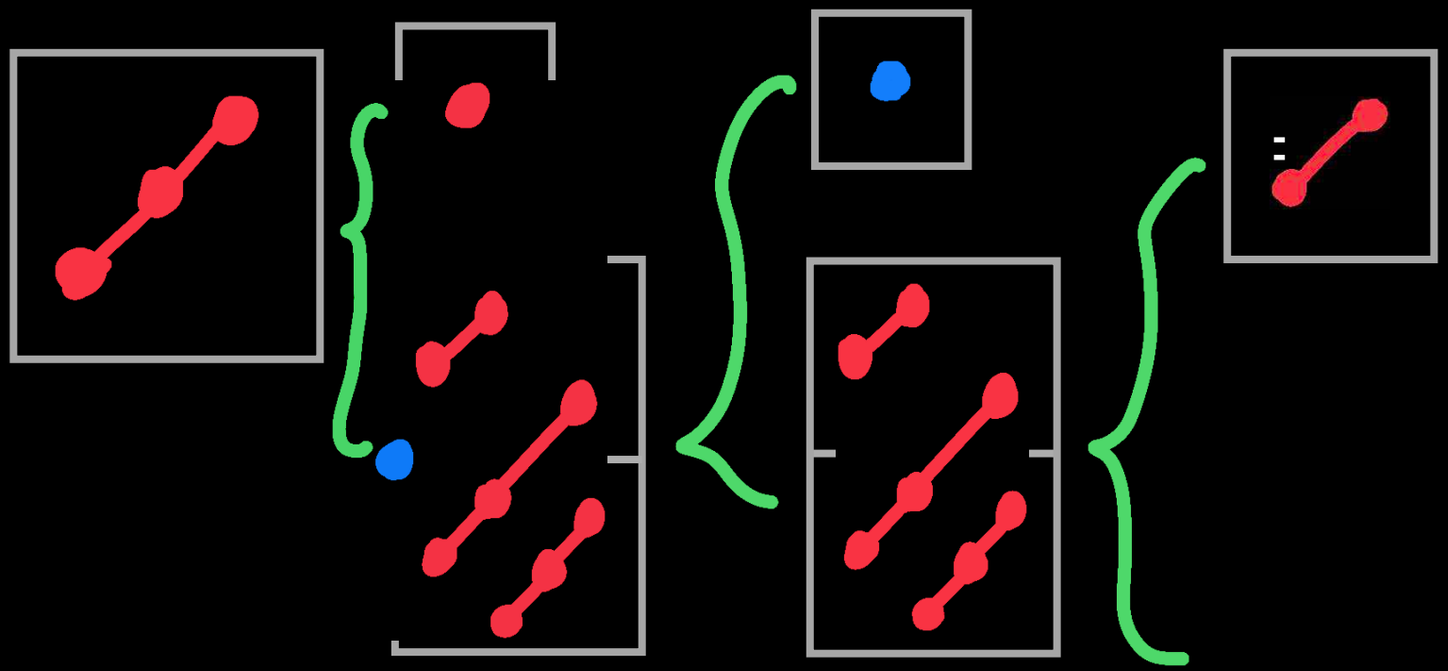


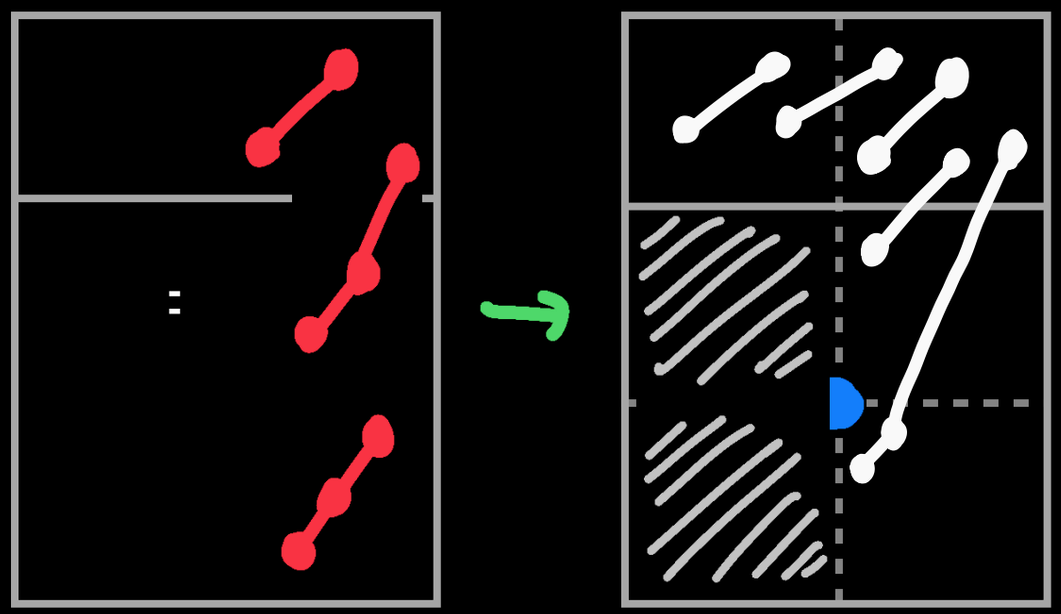


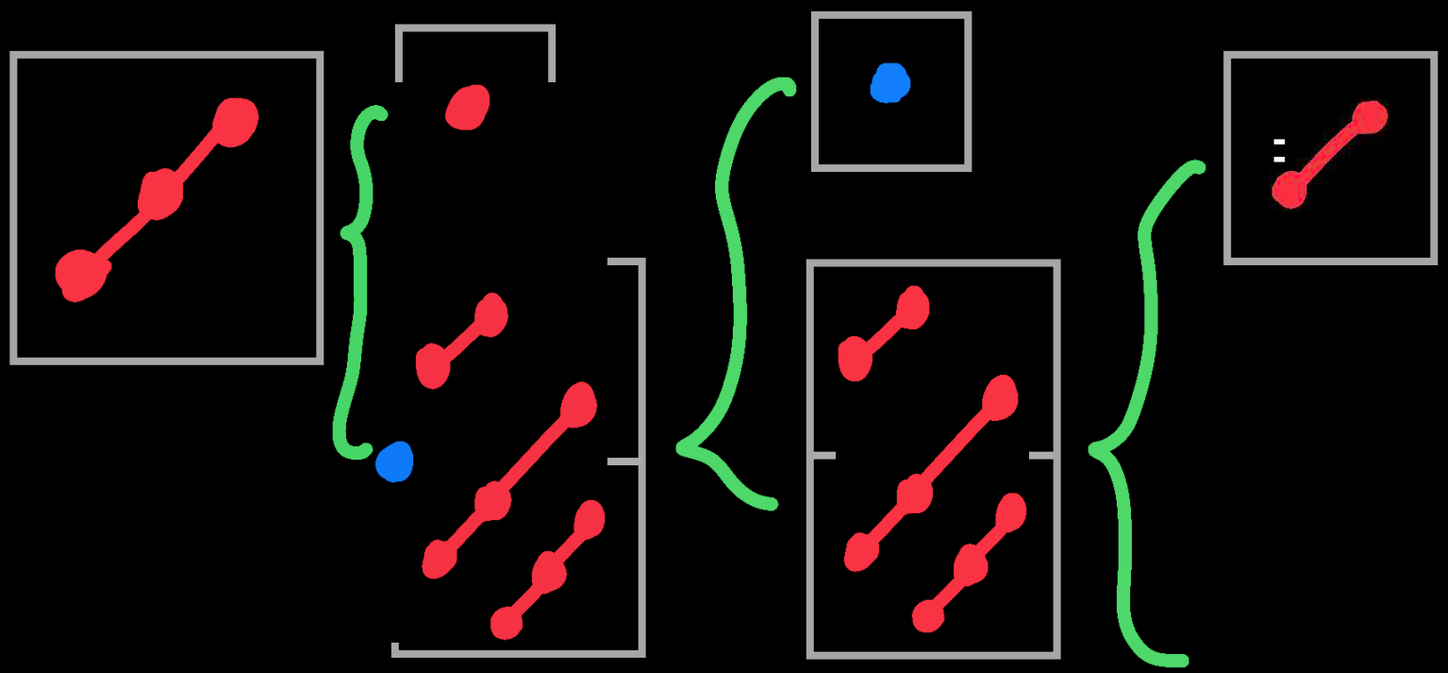


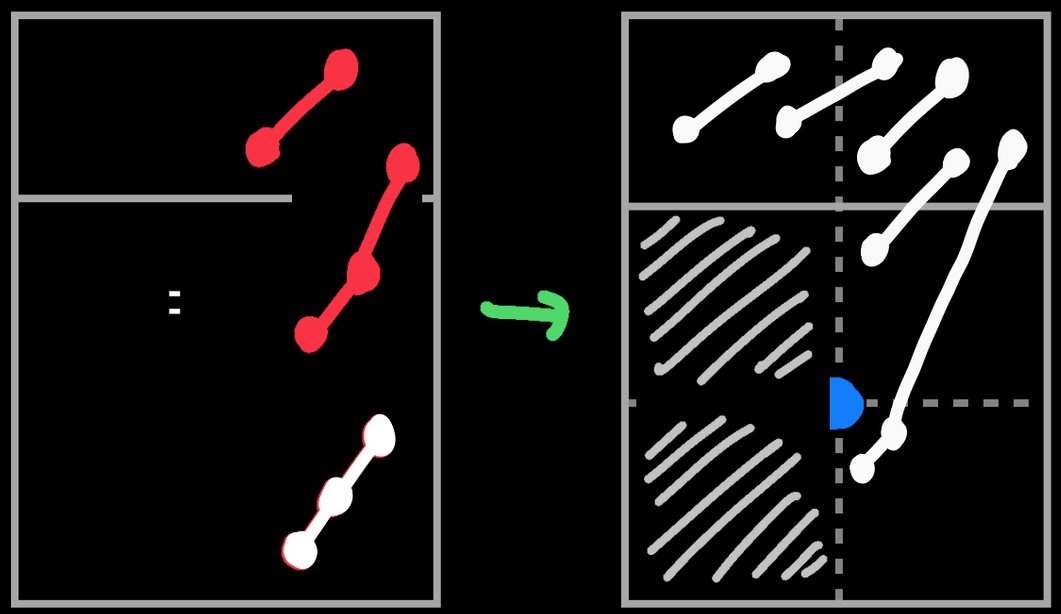


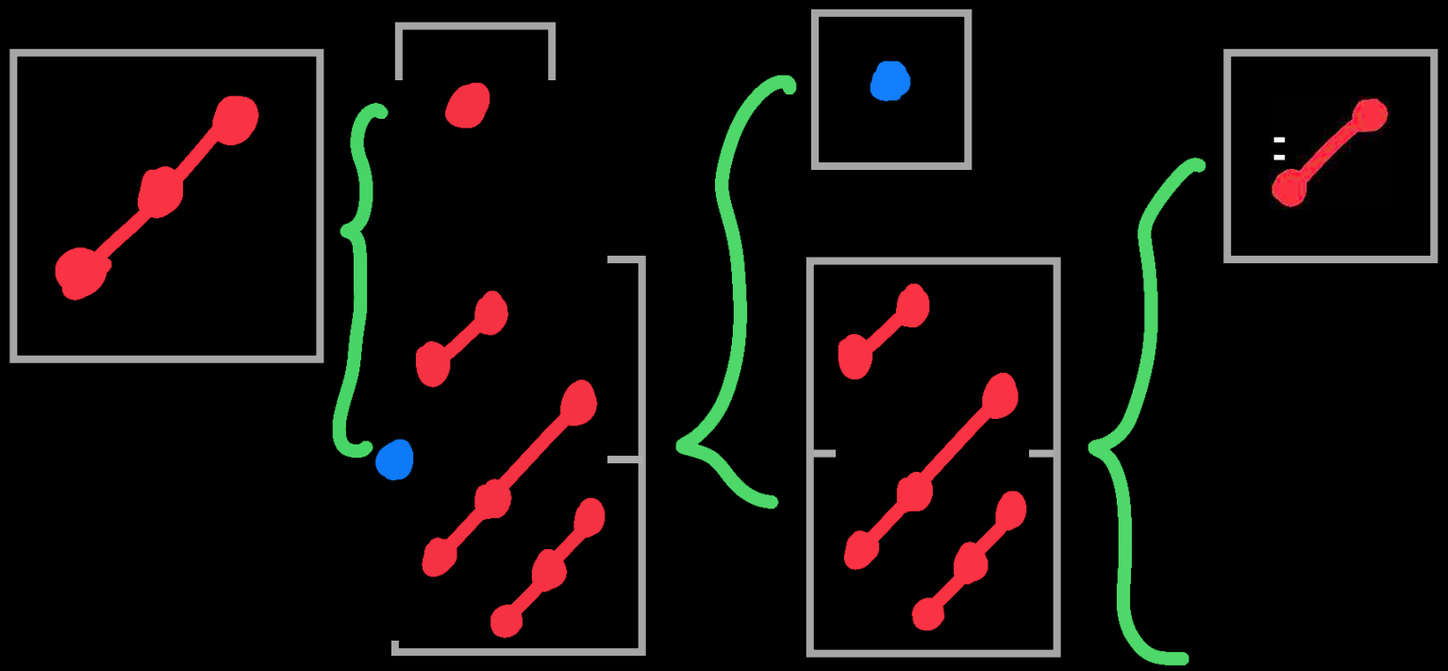


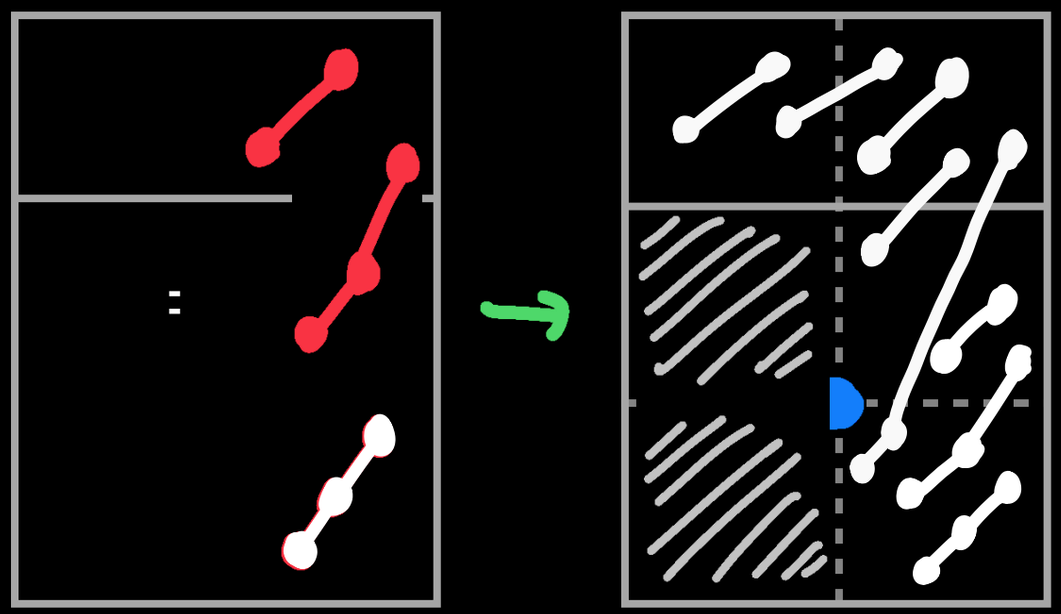


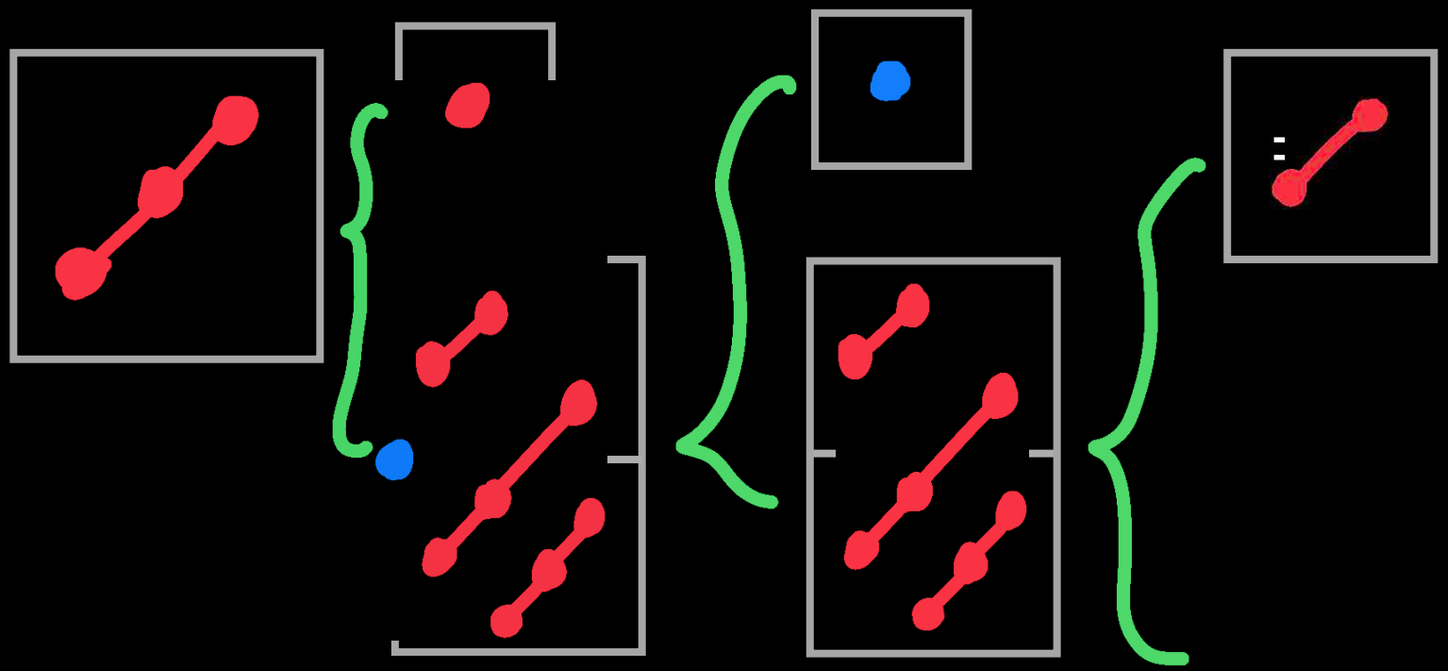


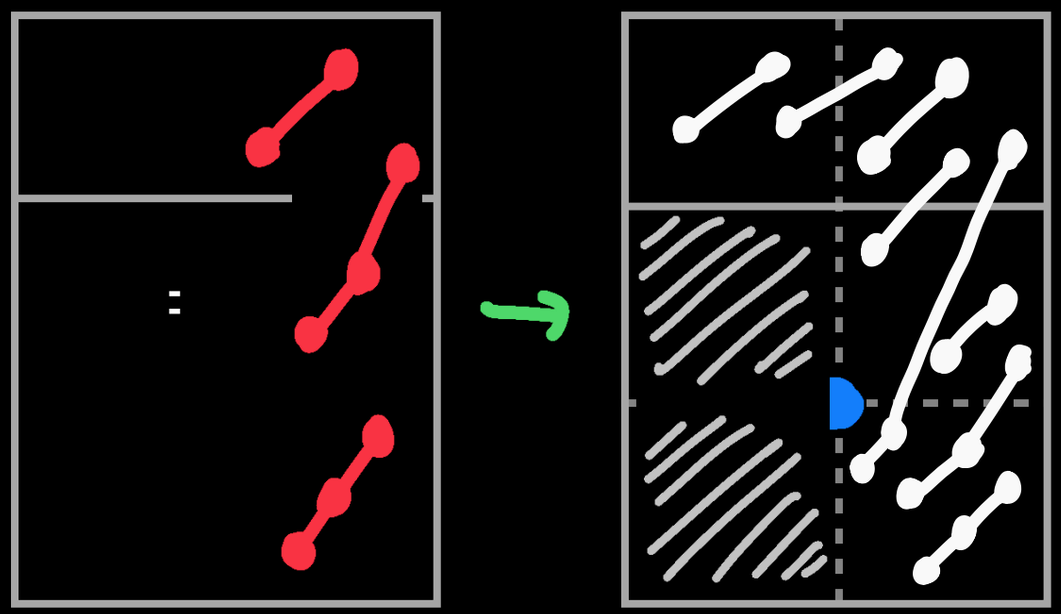


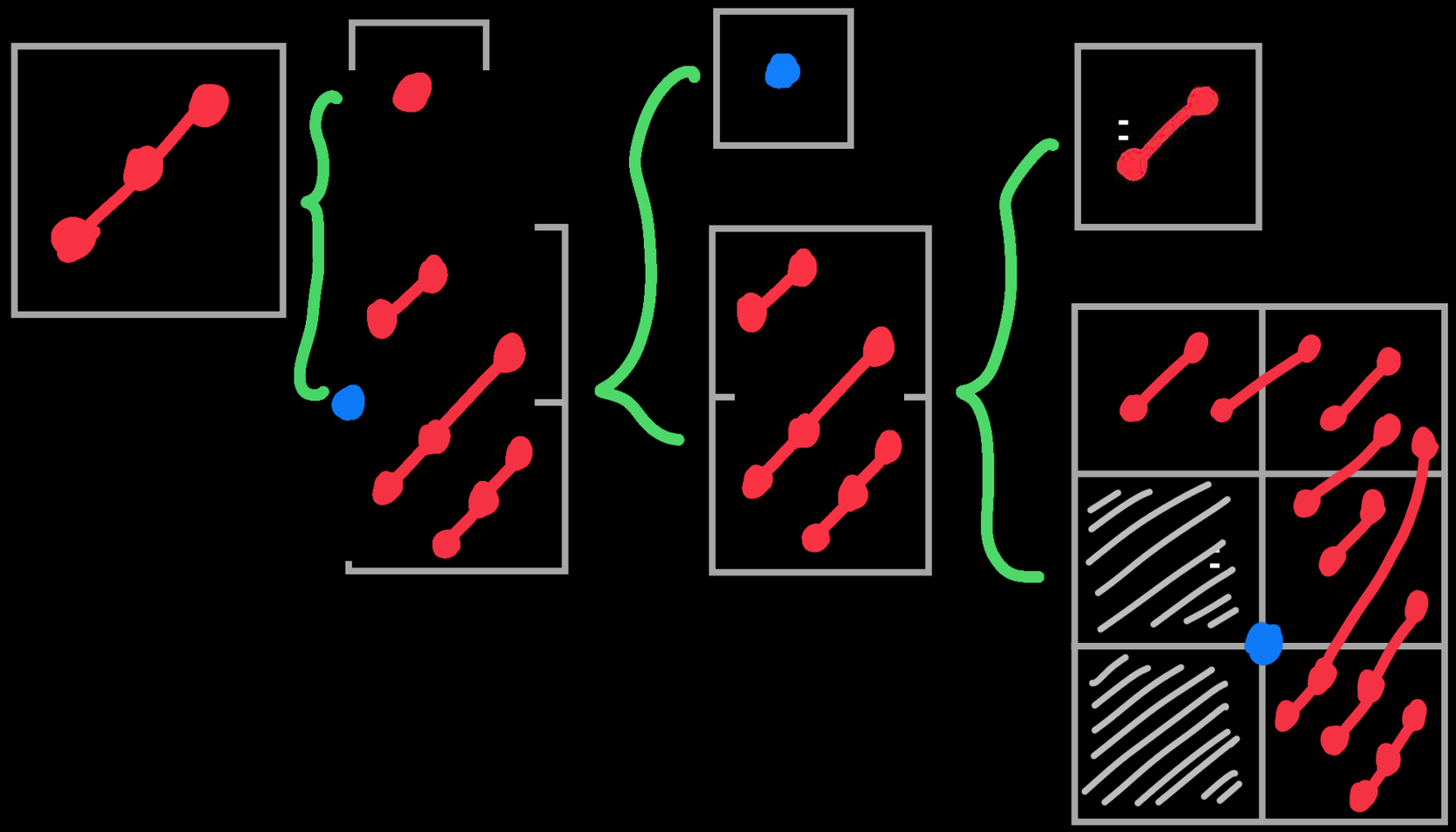


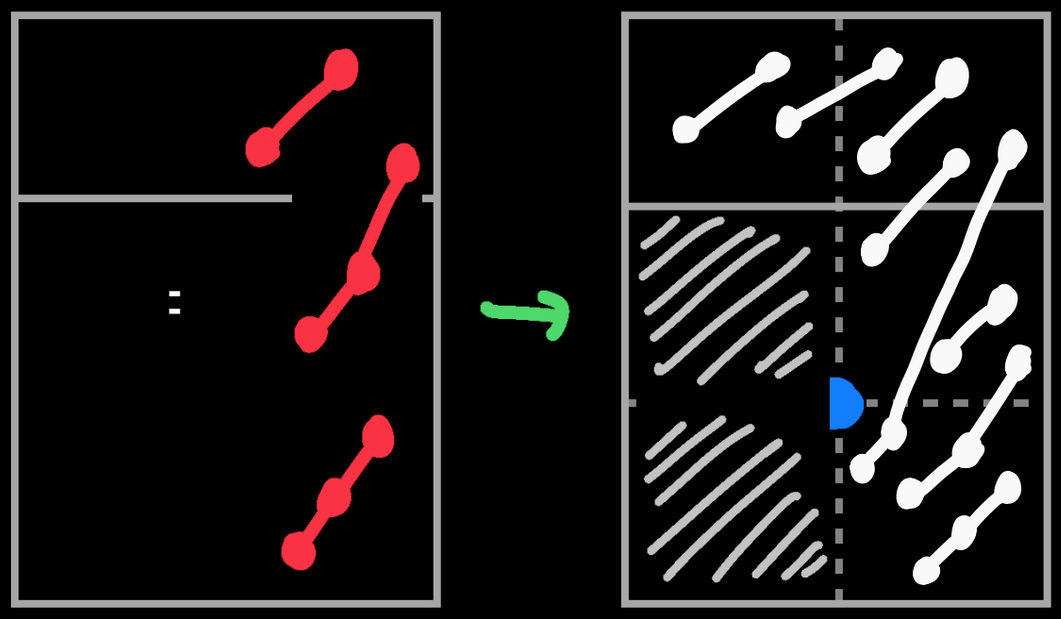


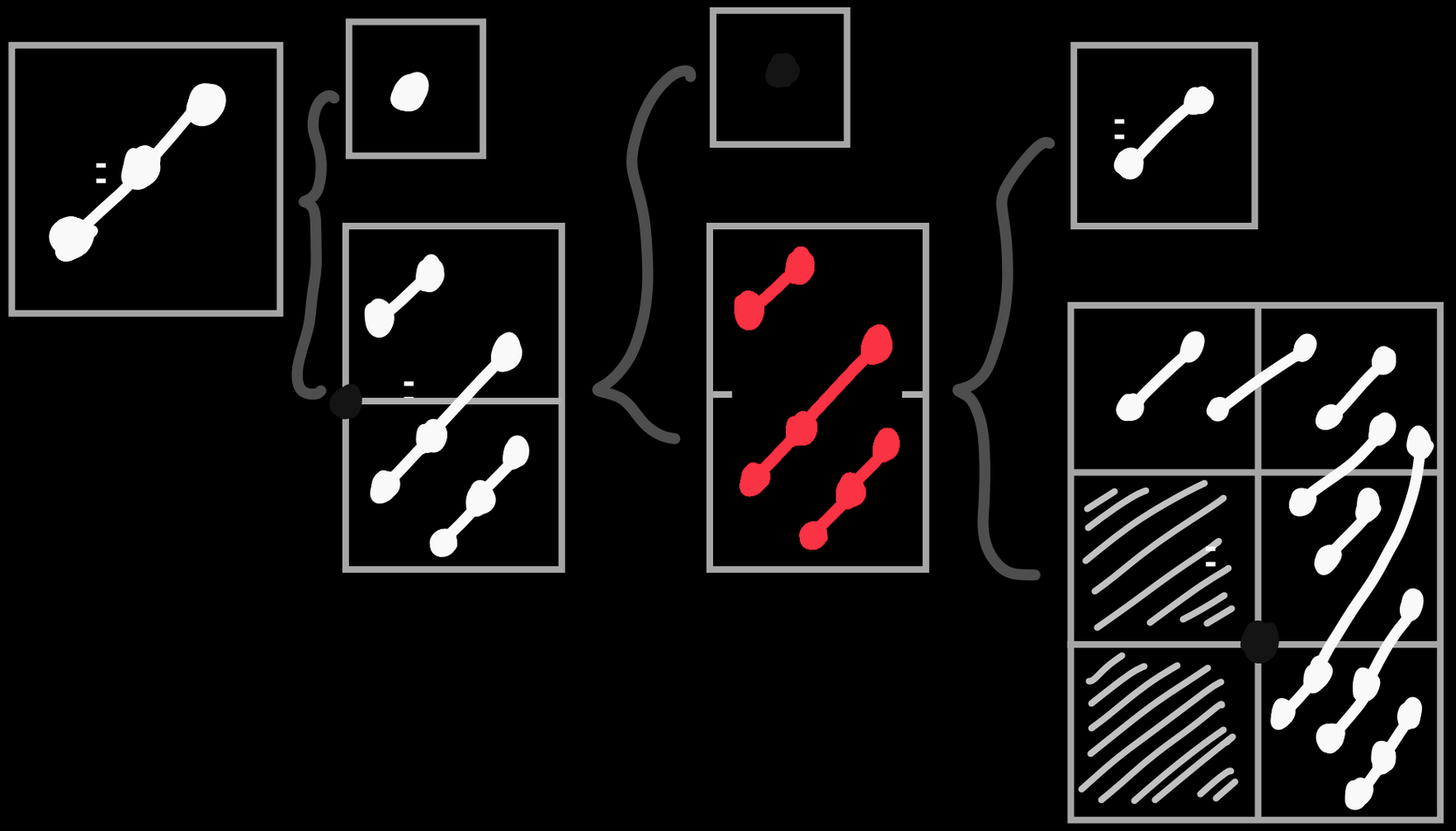


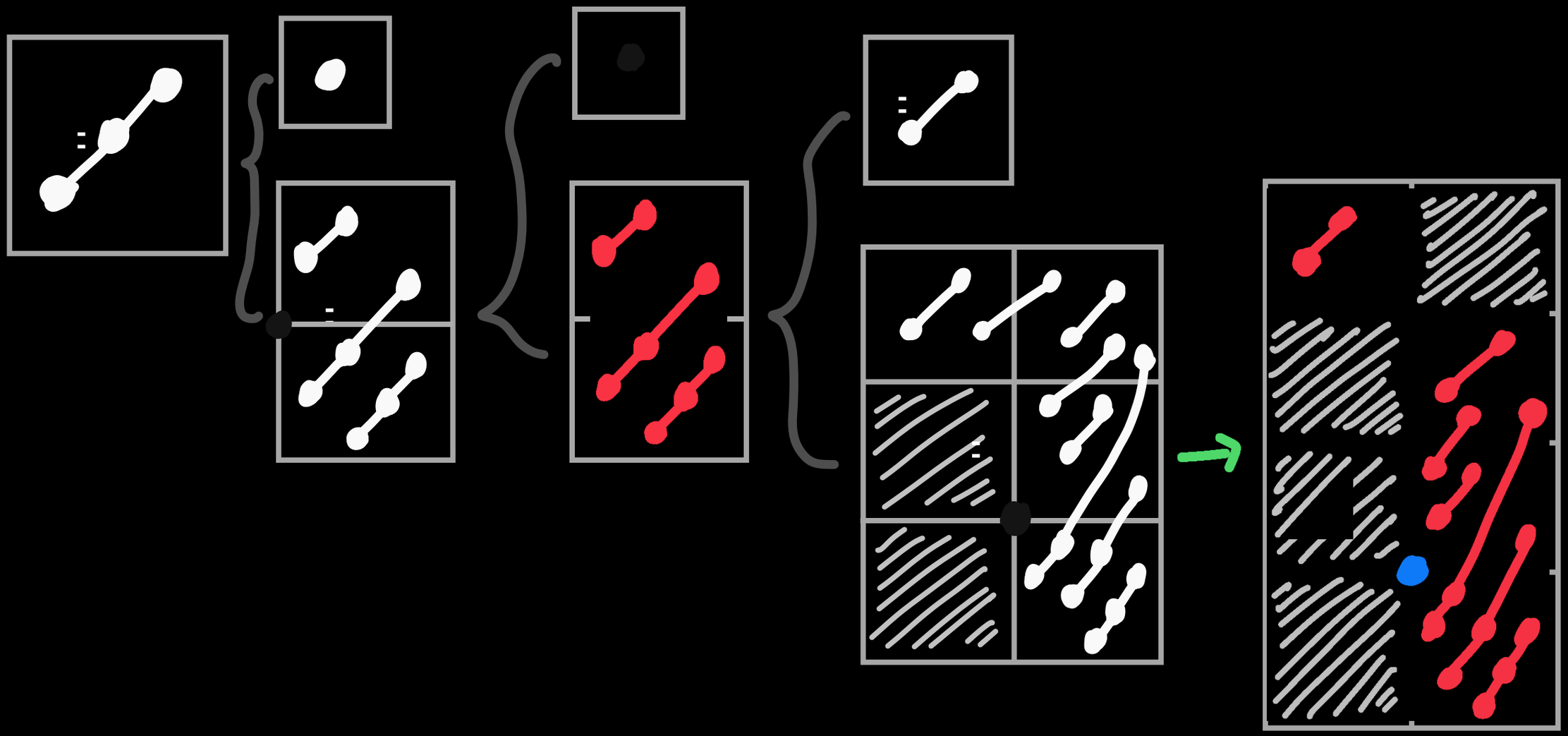


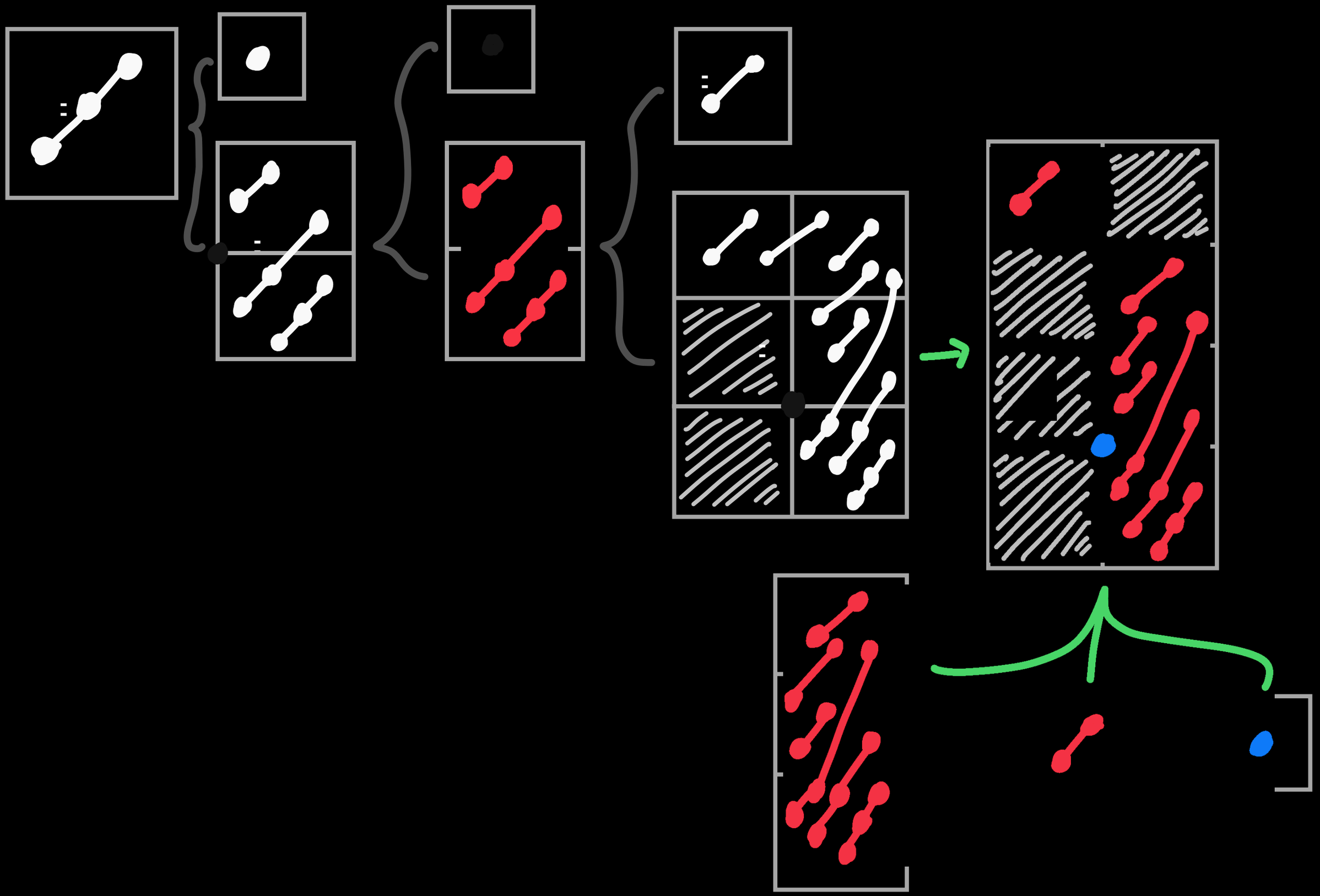


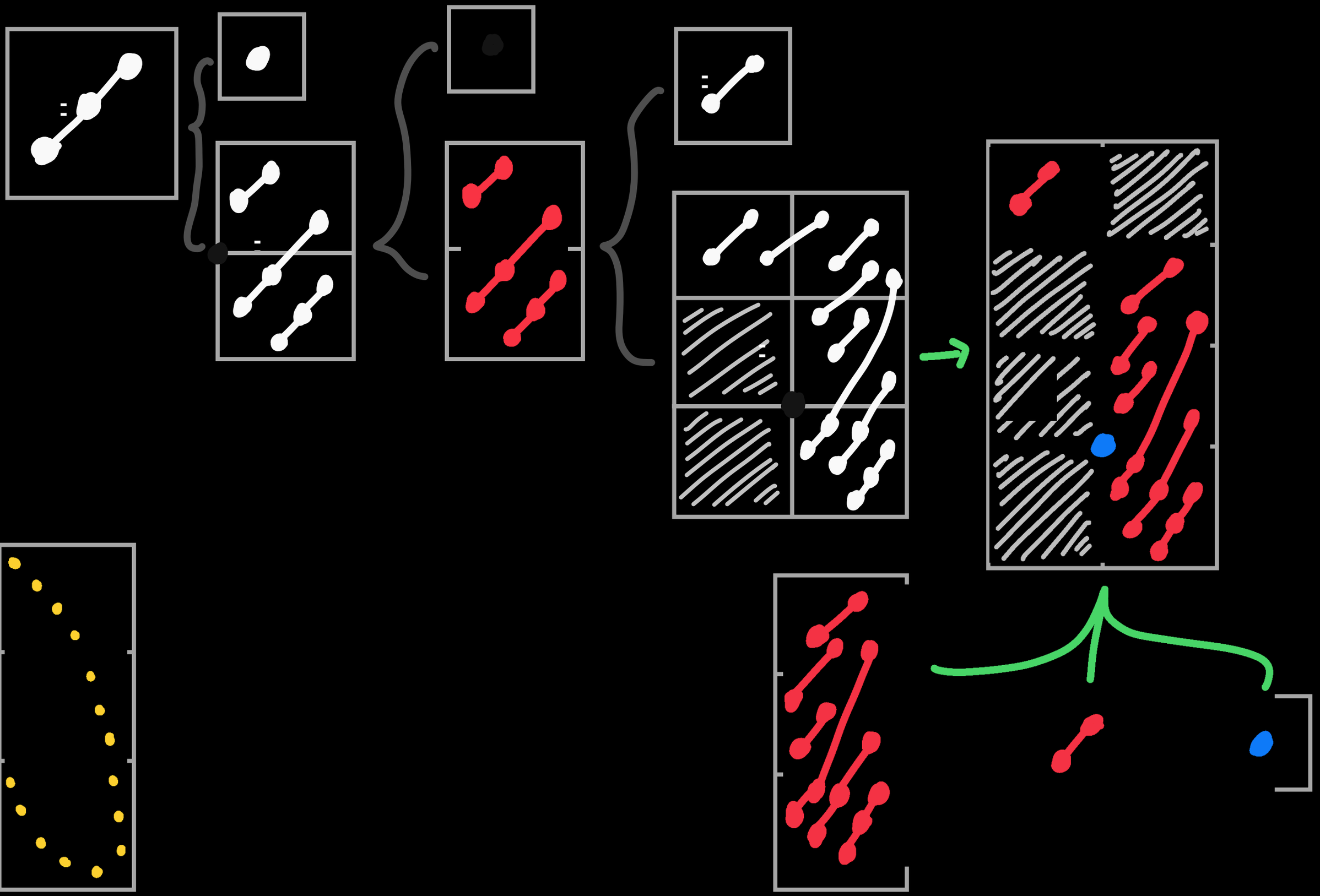


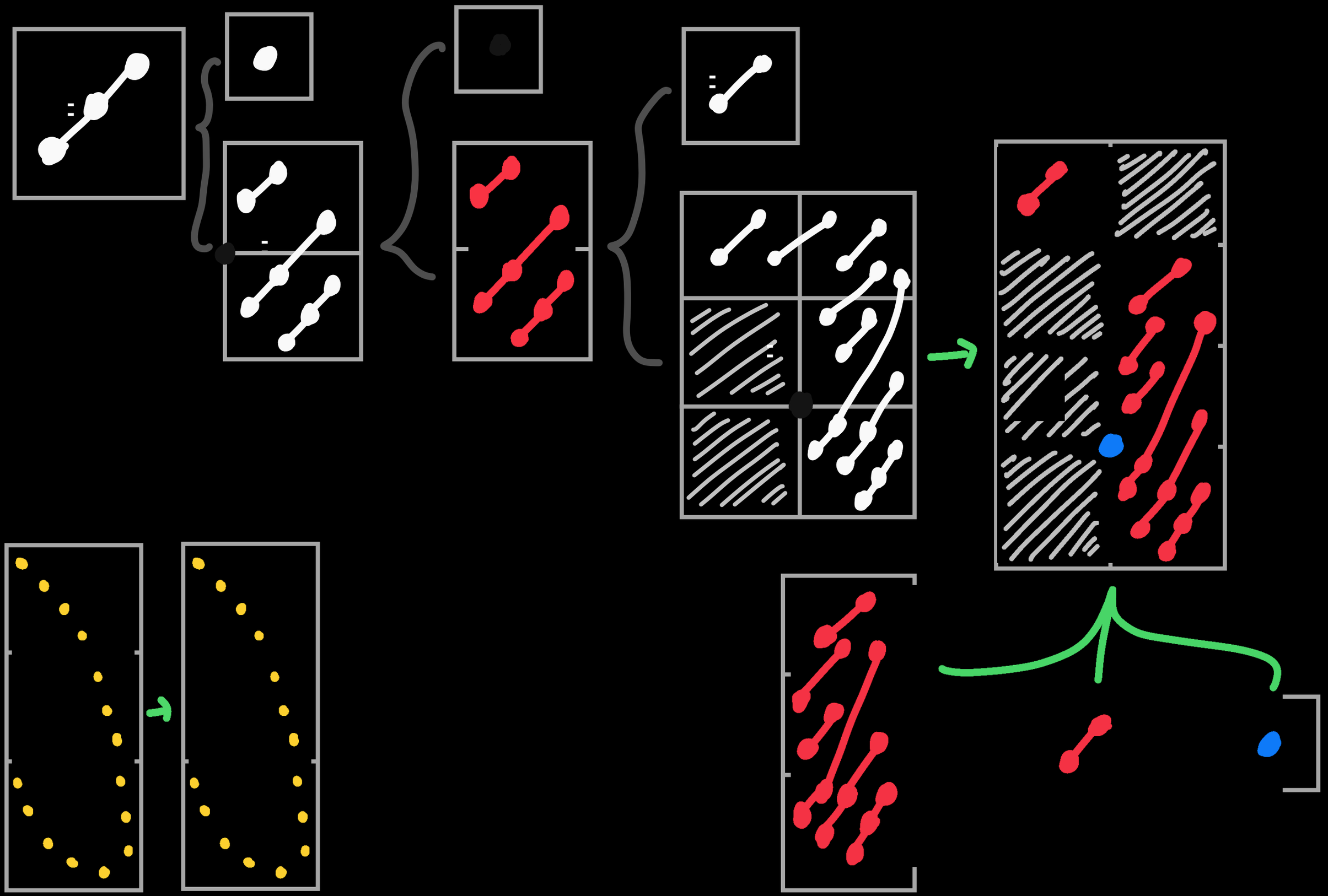


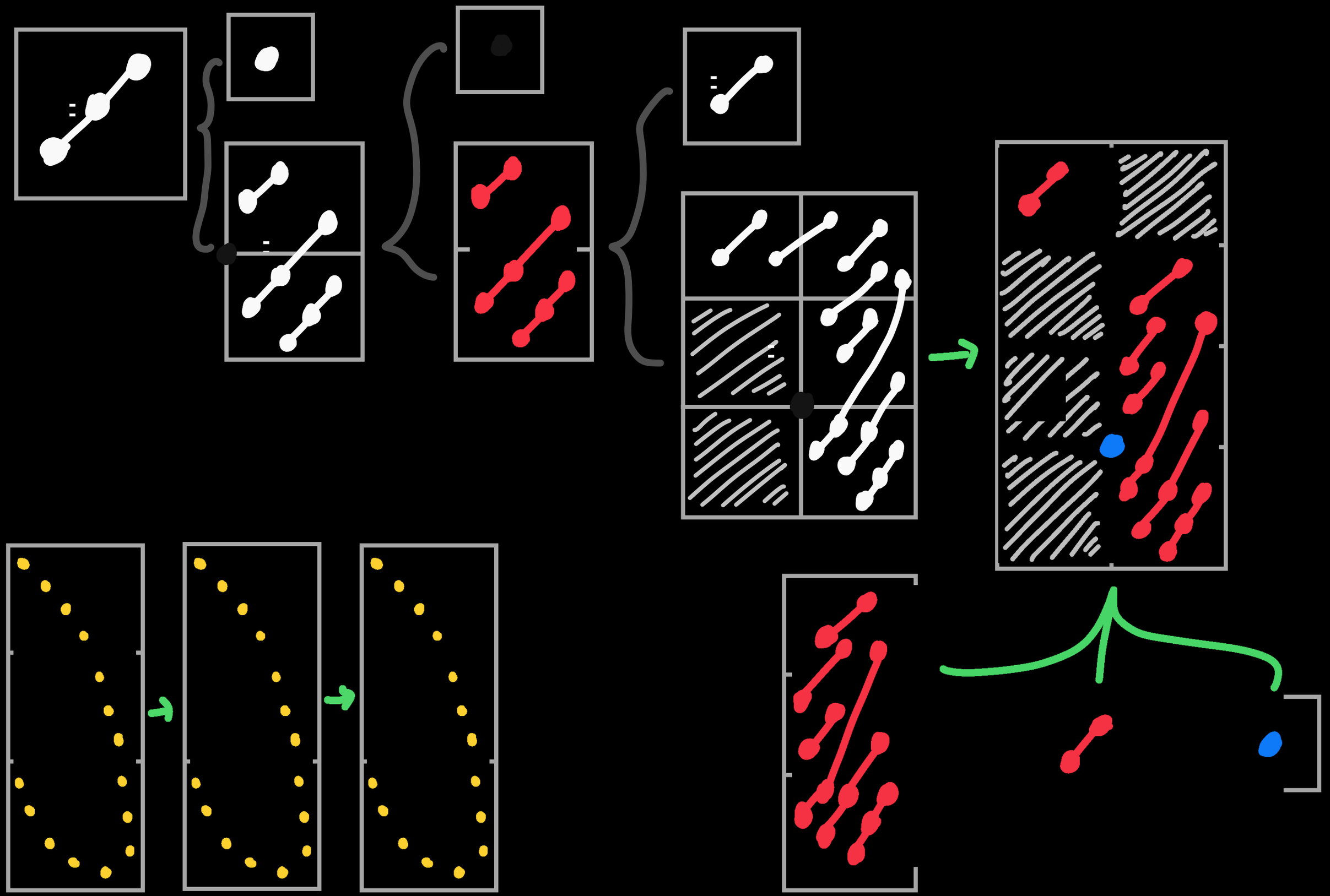


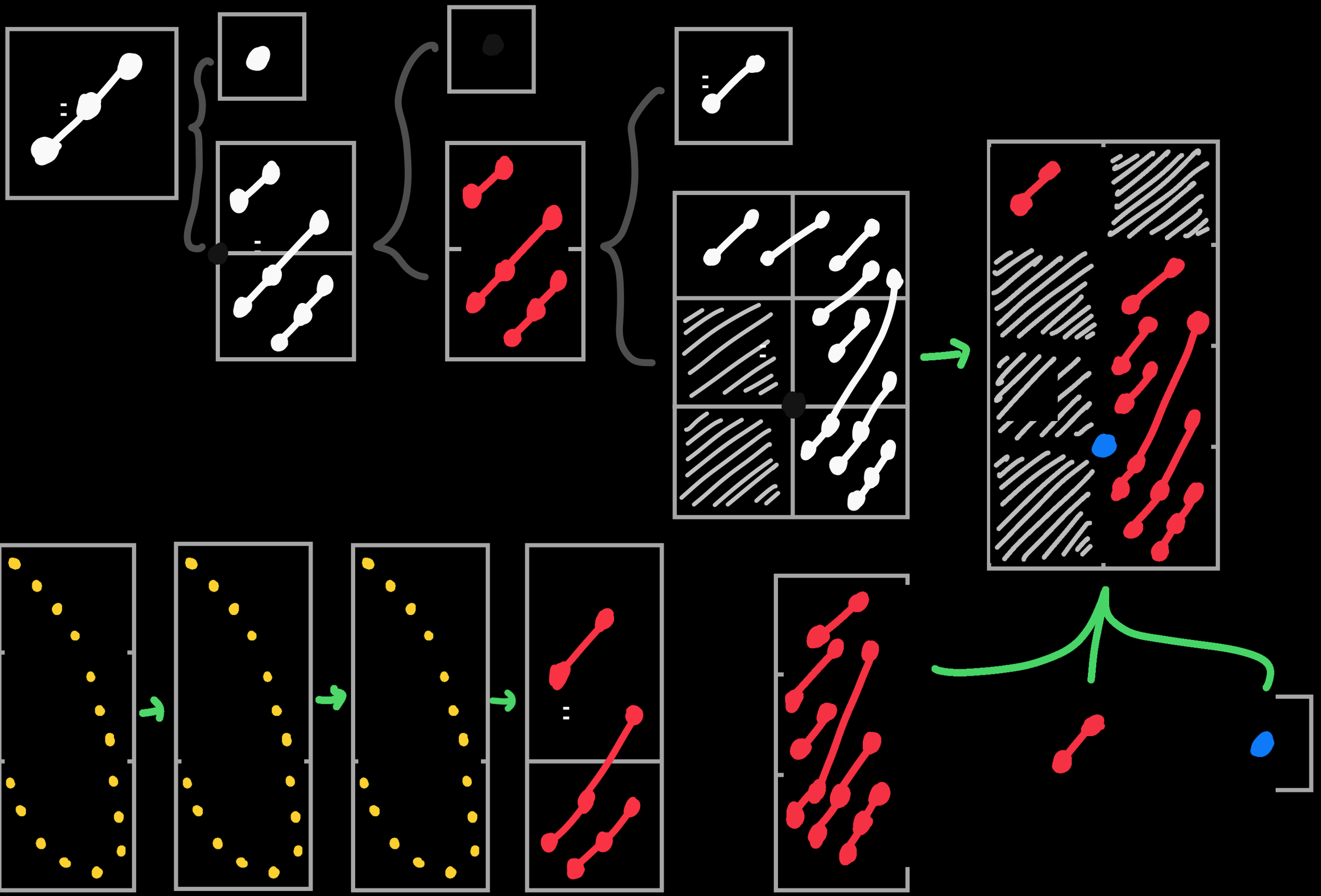


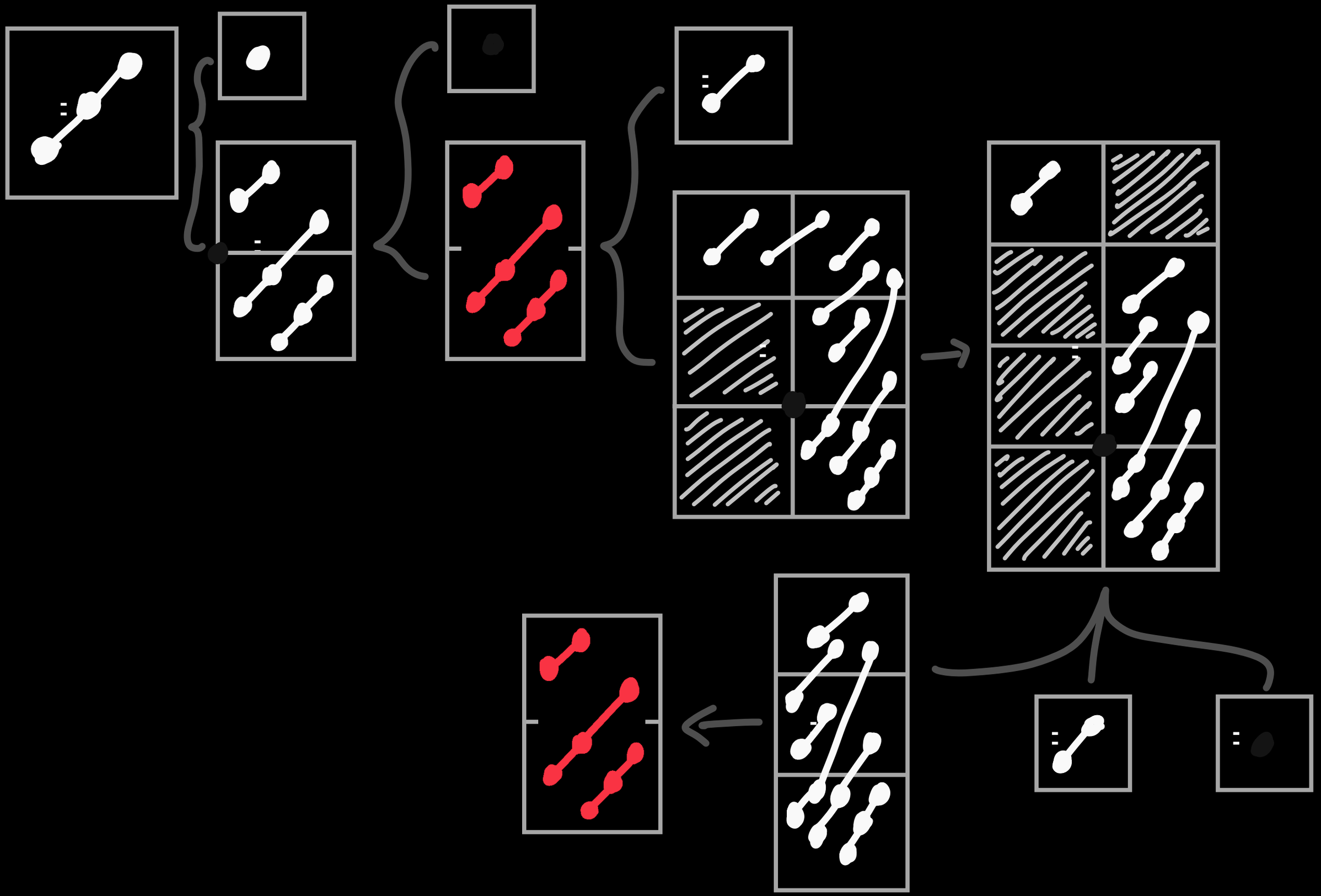


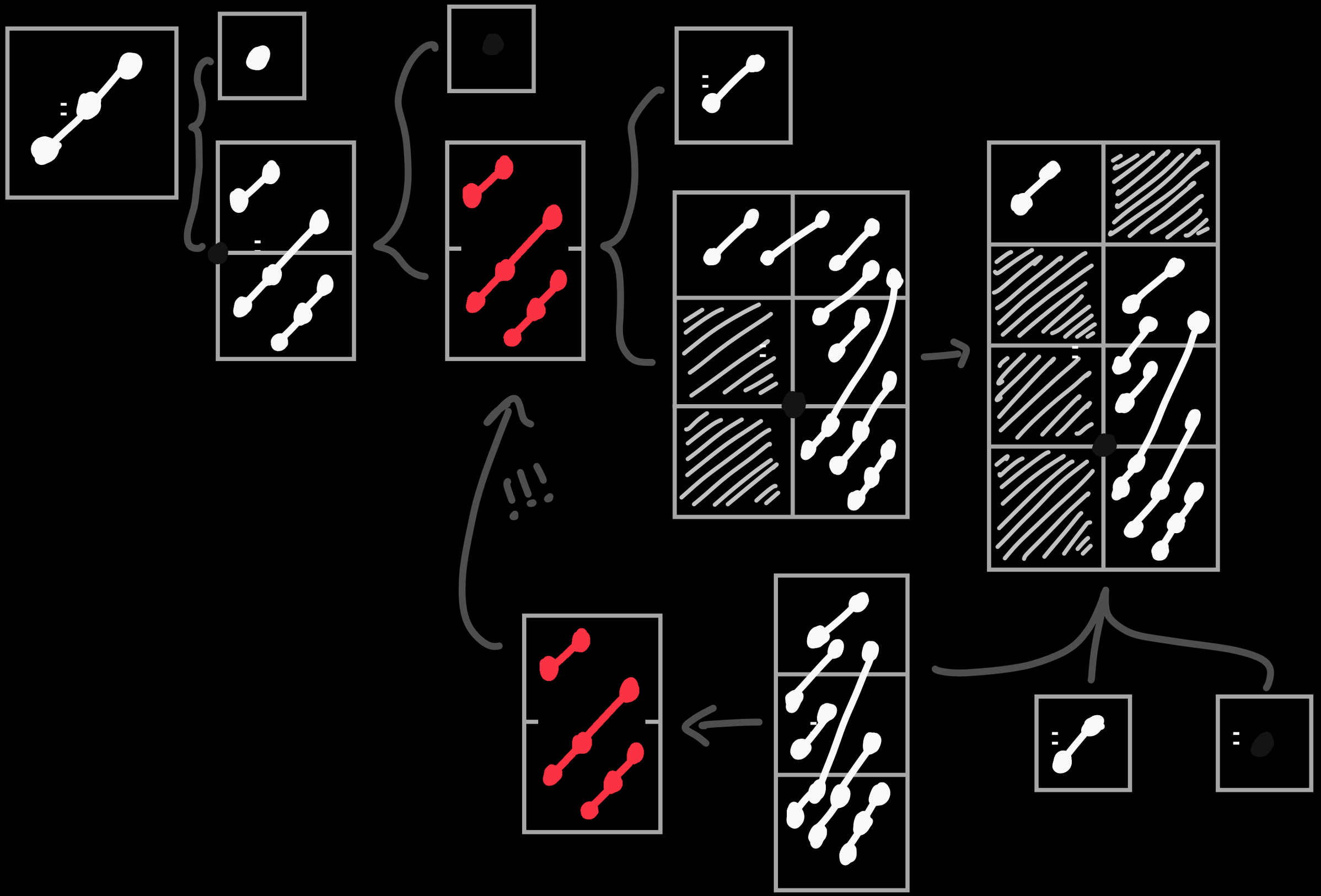


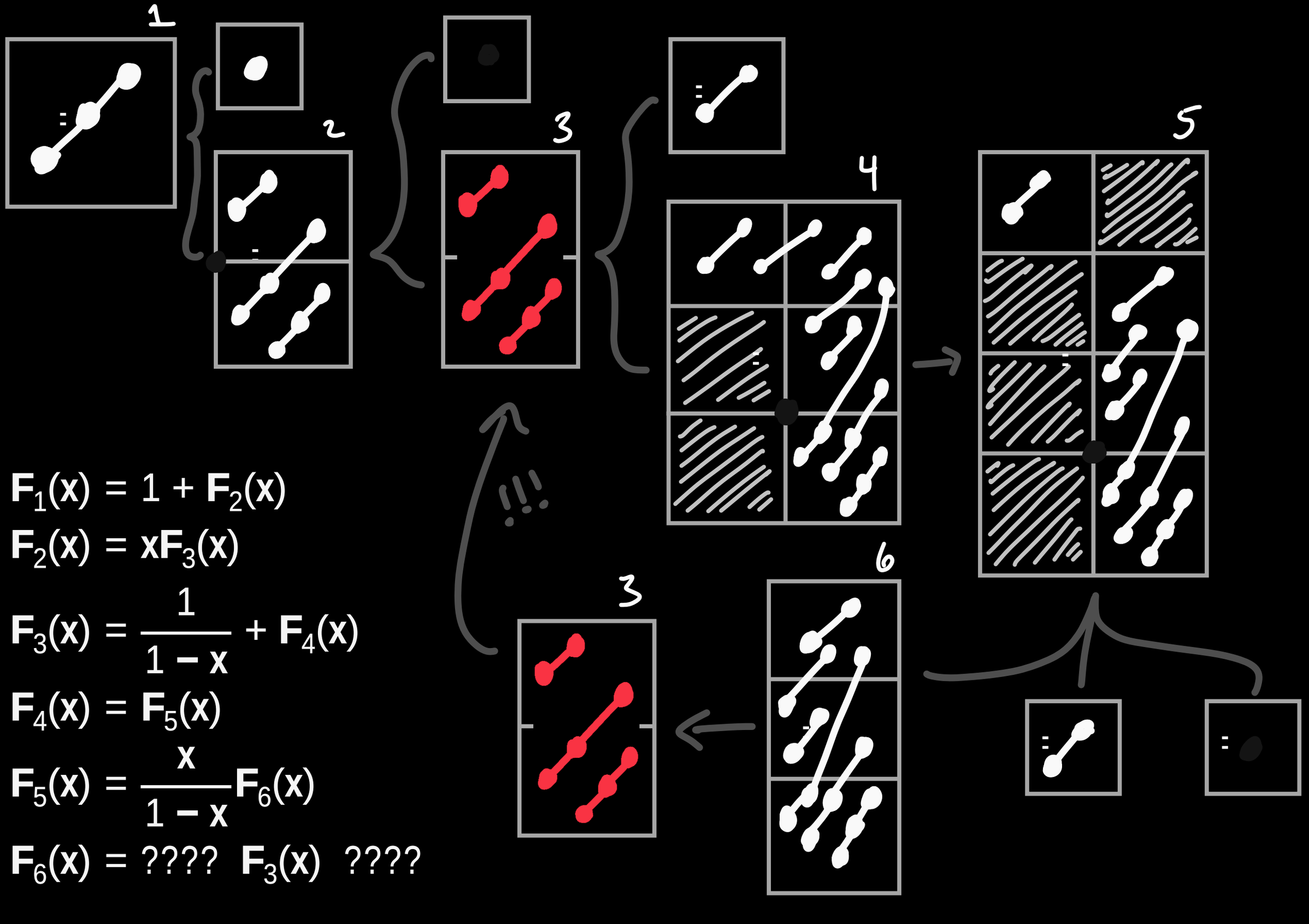












$$F_1(x) = 1 + F_2(x)$$

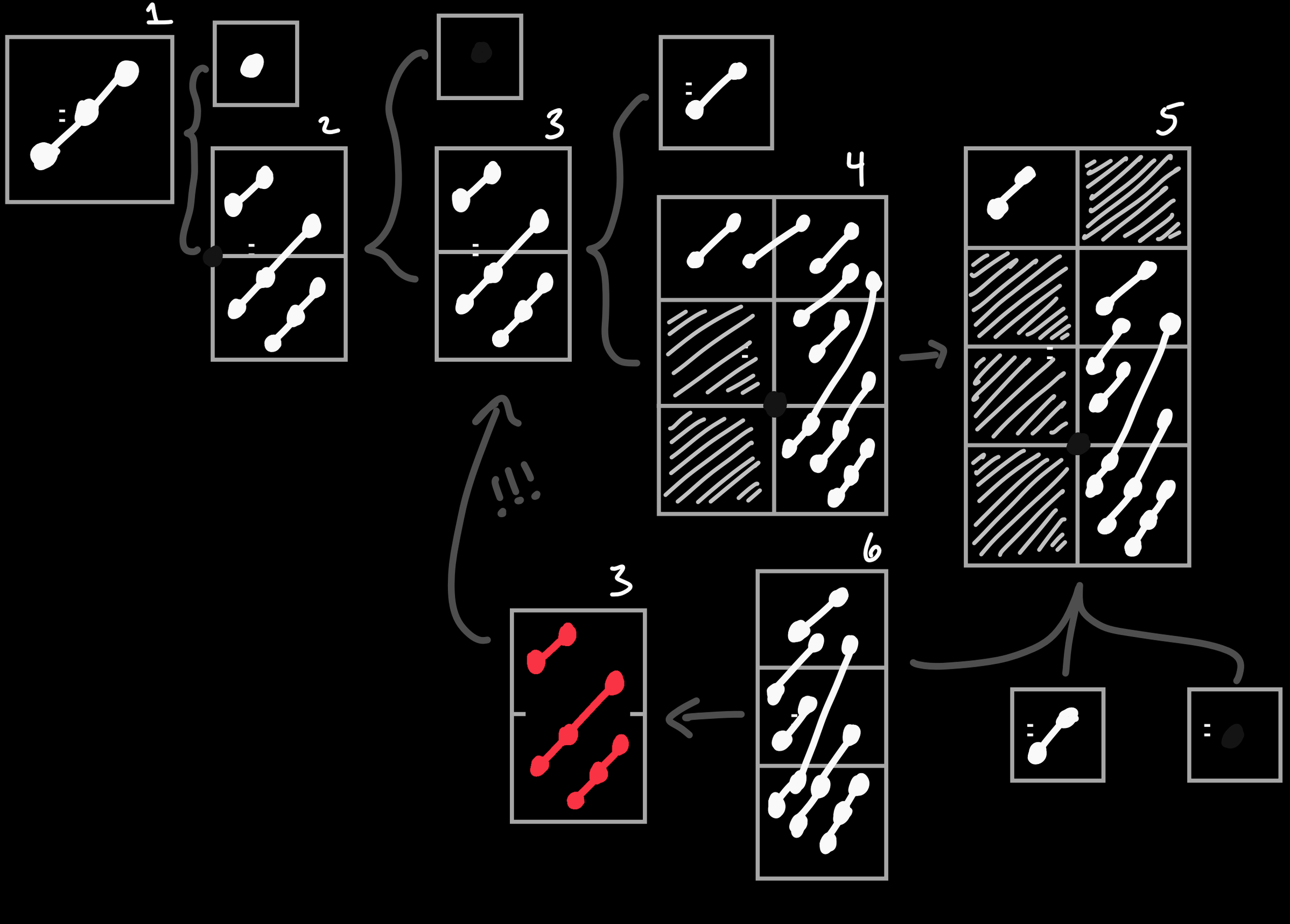
$$F_2(x) = xF_3(x)$$

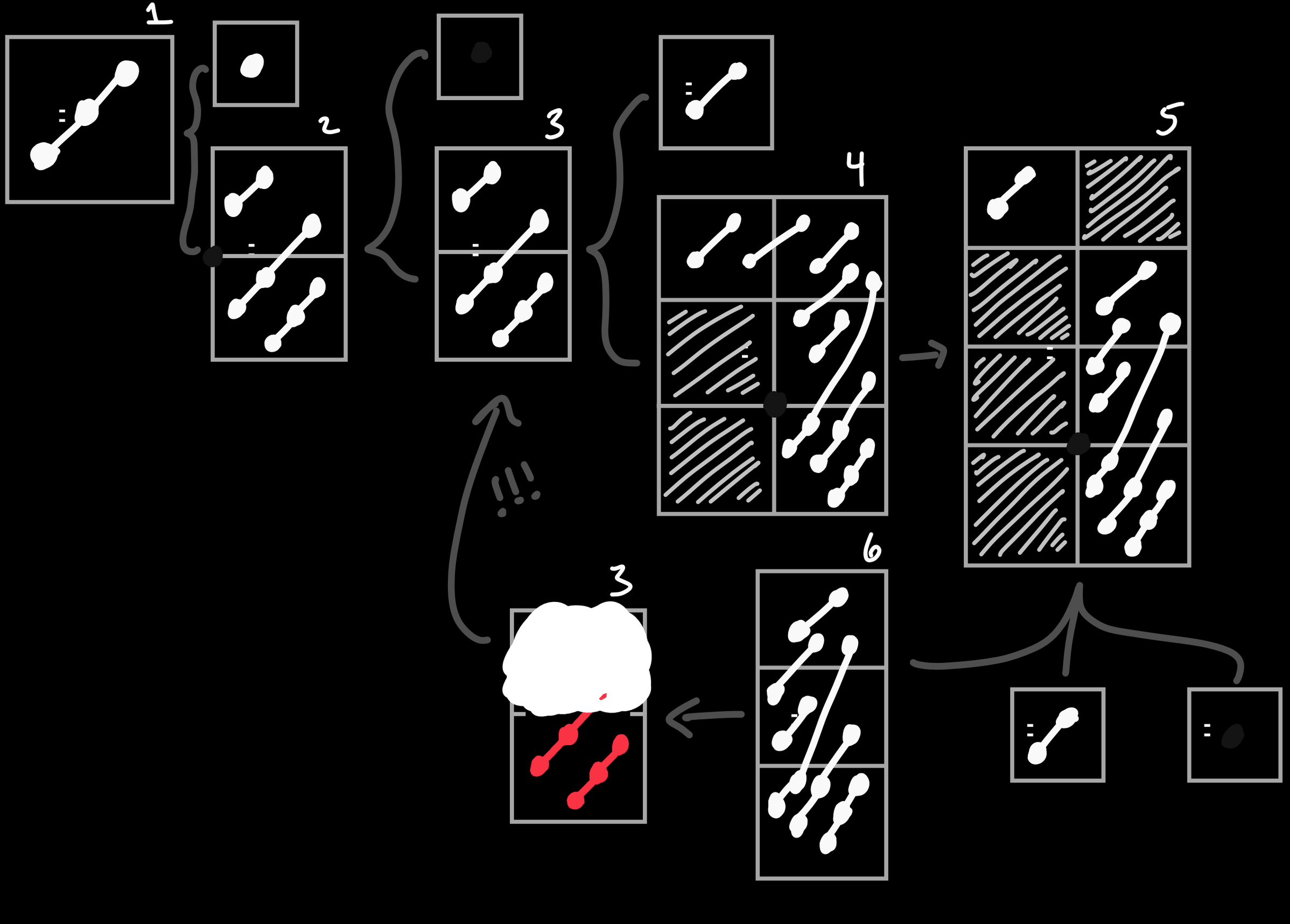
$$F_3(x) = \frac{1}{1-x} + F_4(x)$$

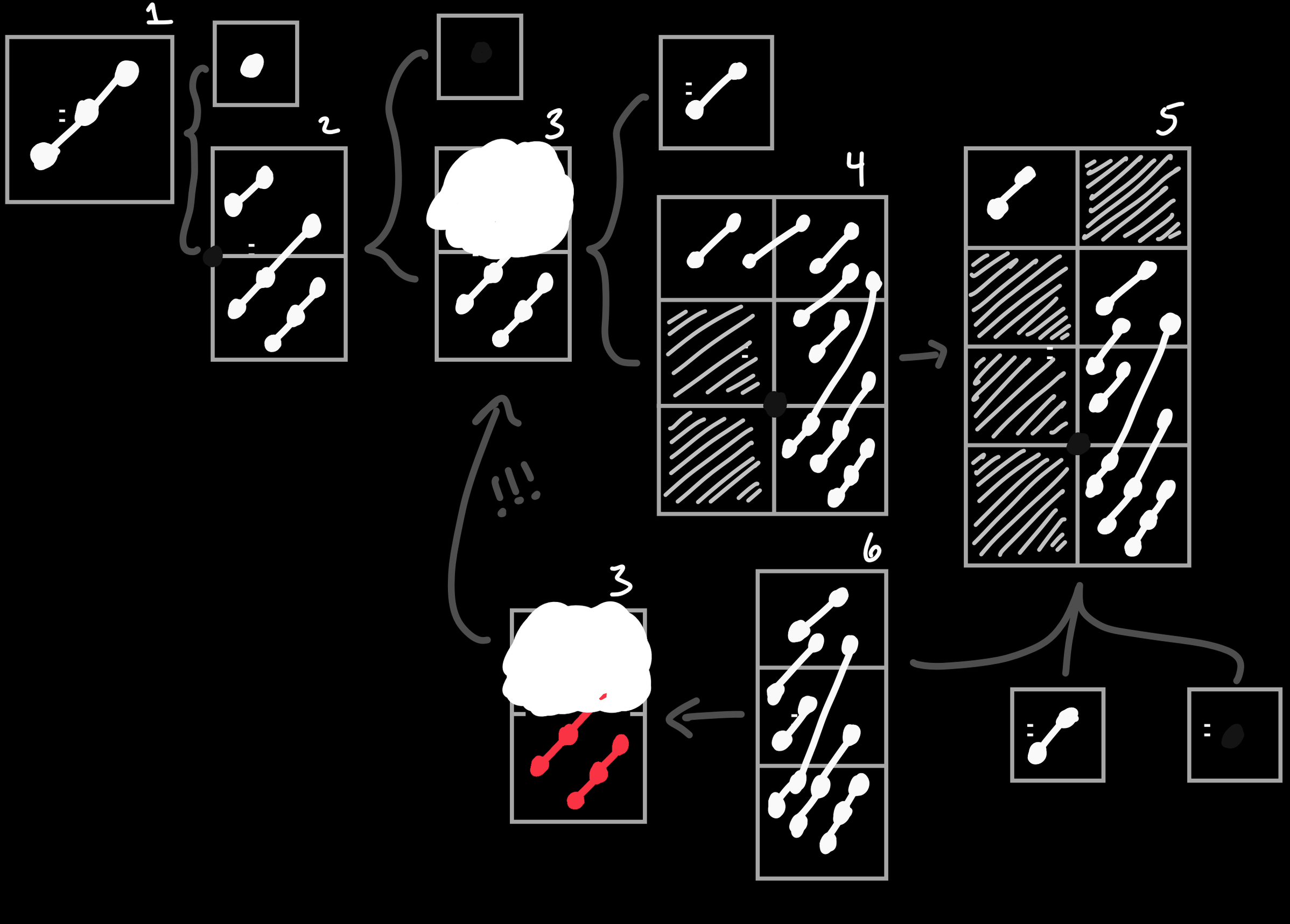
$$F_4(x) = F_5(x)$$

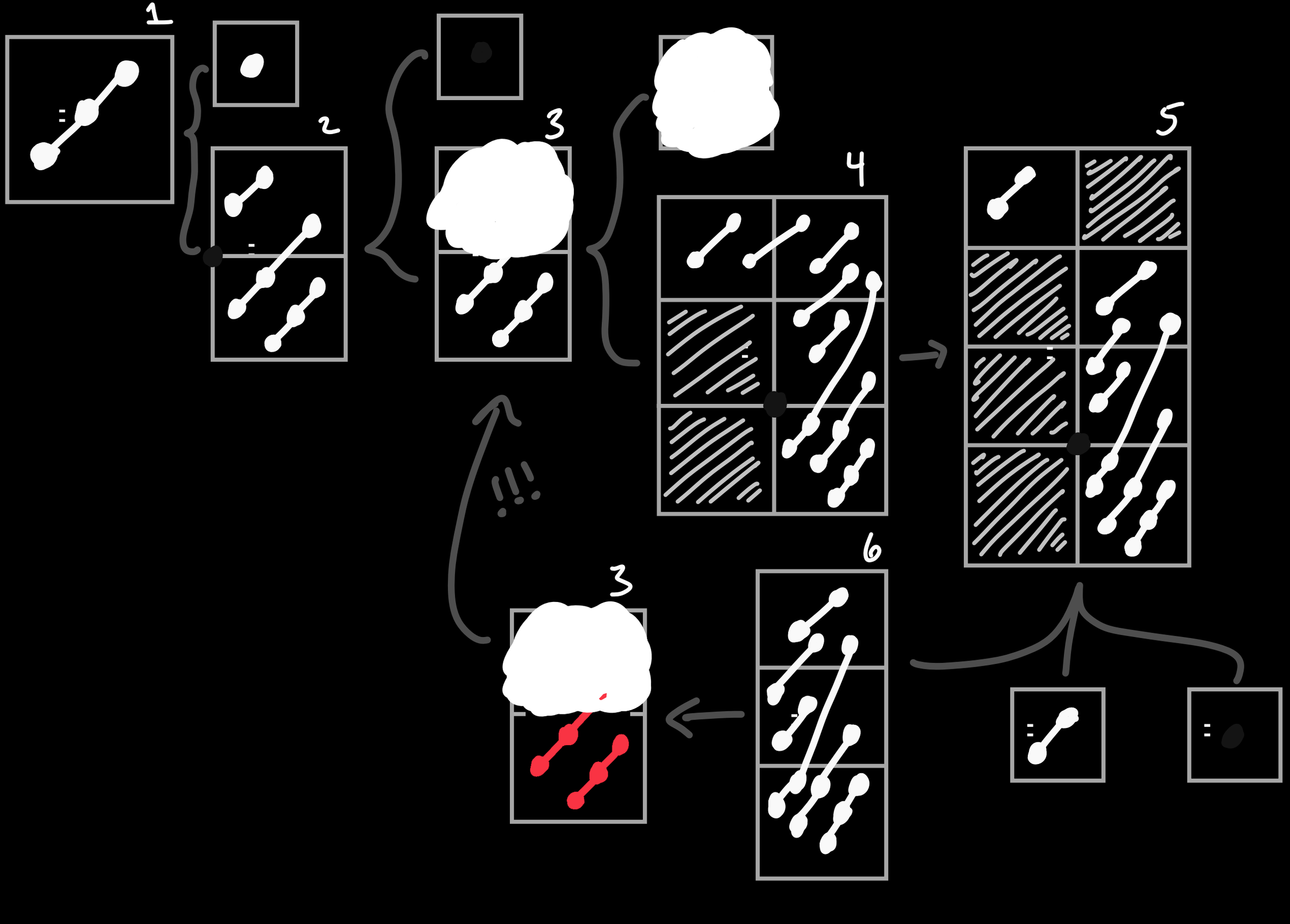
$$F_5(x) = \frac{x}{1-x}F_6(x)$$

$$F_6(x) = ????\ \ F_3(x)\ \ ????$$


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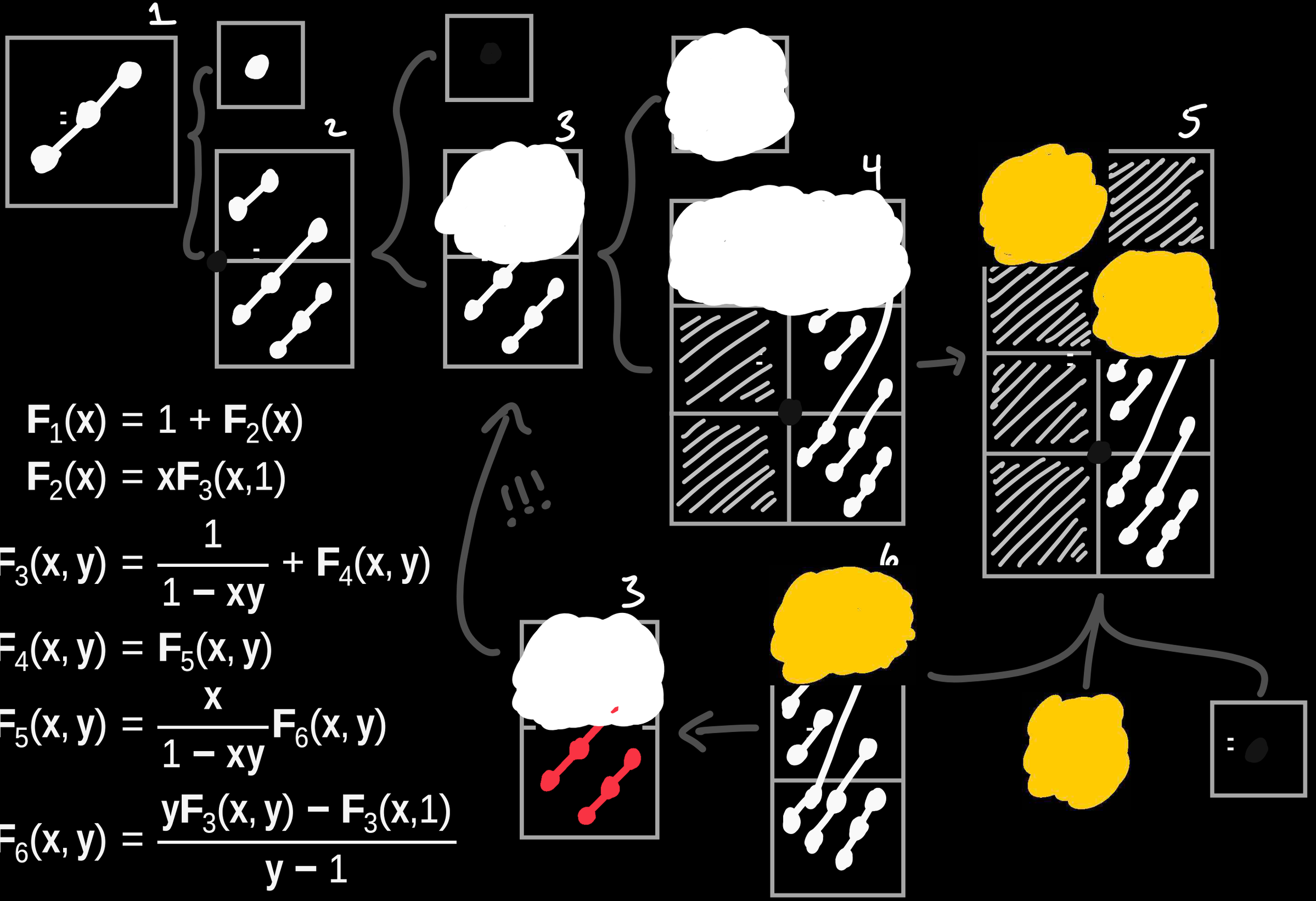

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$$F_1(x) = 1 + F_2(x)$$

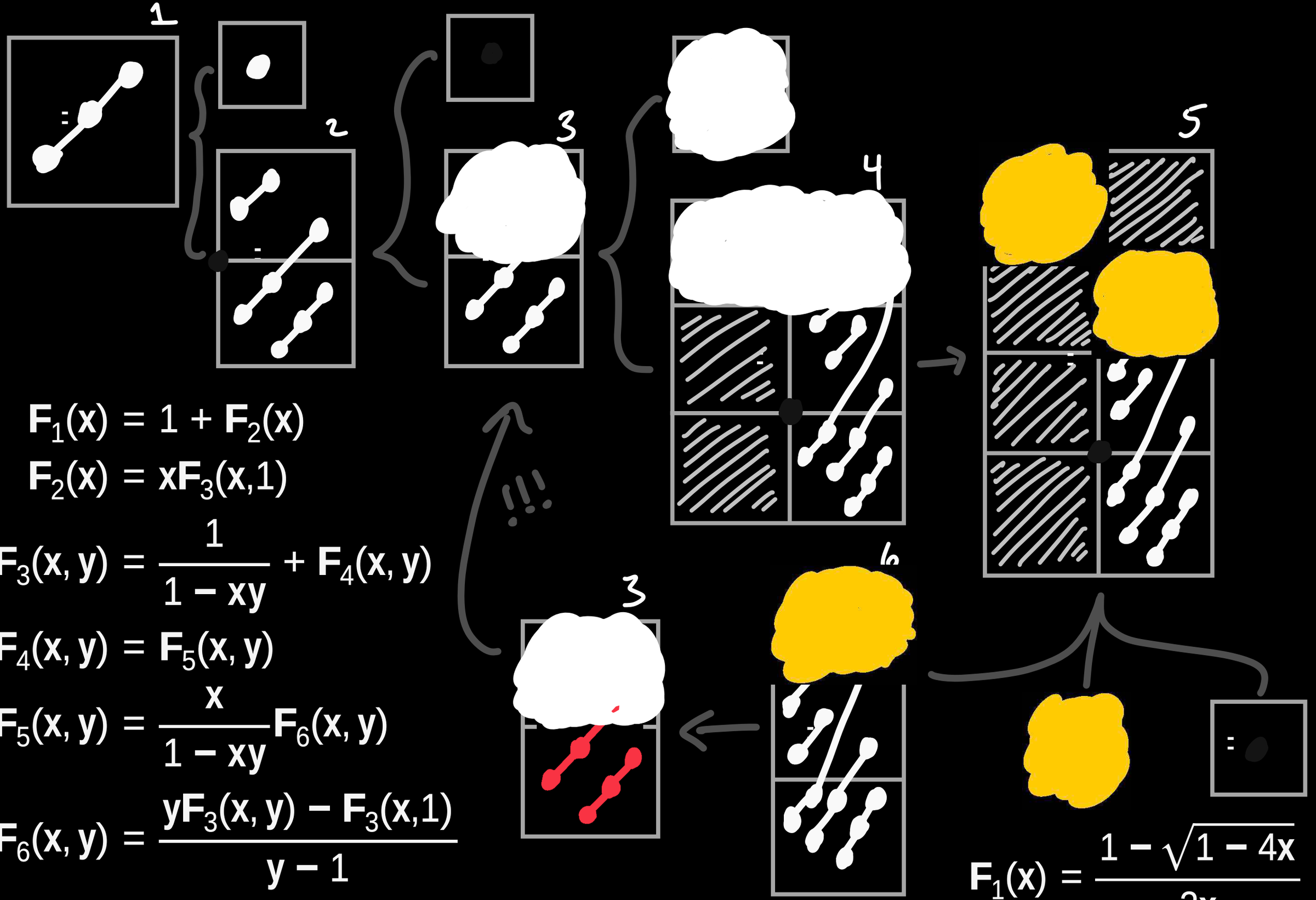
$$F_2(x) = xF_3(x,1)$$

$$F_3(x, y) = \frac{1}{1 - xy} + F_4(x, y)$$

$$F_4(x, y) = F_5(x, y)$$

$$F_5(x, y) = \frac{x}{1 - xy} F_6(x, y)$$

$$F_6(x, y) = \frac{yF_3(x, y) - F_3(x,1)}{y - 1}$$



$$F_1(x) = 1 + F_2(x)$$

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$$F_6(x, y) = \frac{yF_3(x, y) - F_3(x,1)}{y - 1}$$

$$F_1(x) = \frac{1 - \sqrt{1 - 4x}}{2x}$$

NO

qfibp`lmb

So we have:

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Tilings = sets of permutations



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Strategies = rigorous deductions that some tilings can be decomposed in terms of others

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Proof Trees = a bunch of tilings, related by strategies that are sufficient to derive enumerations, generating functions, etc

(= combinatorial specifications)

So we have:

Tilings = sets of permutations

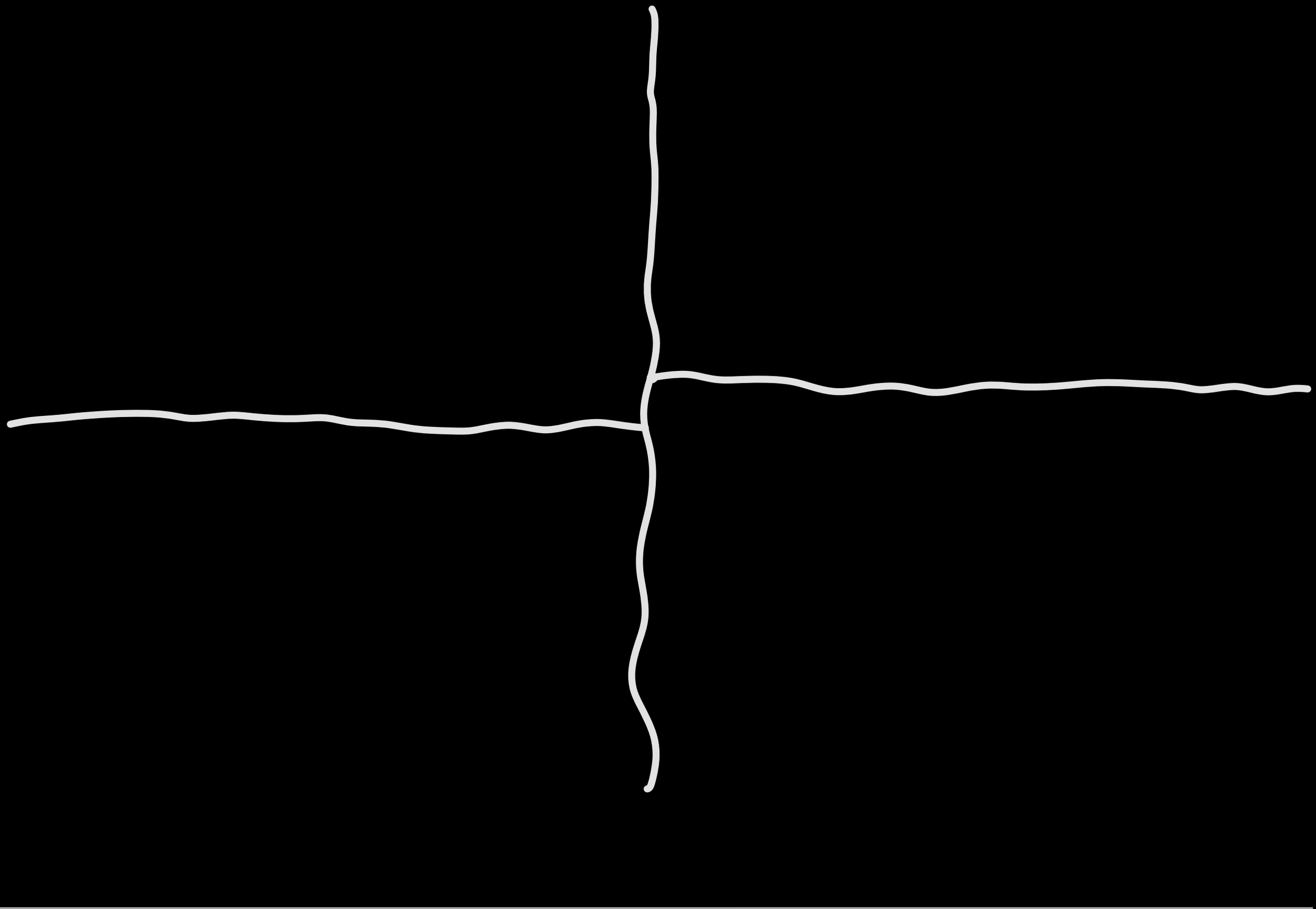


Strategies = rigorous deductions that some tilings can be decomposed in terms of others

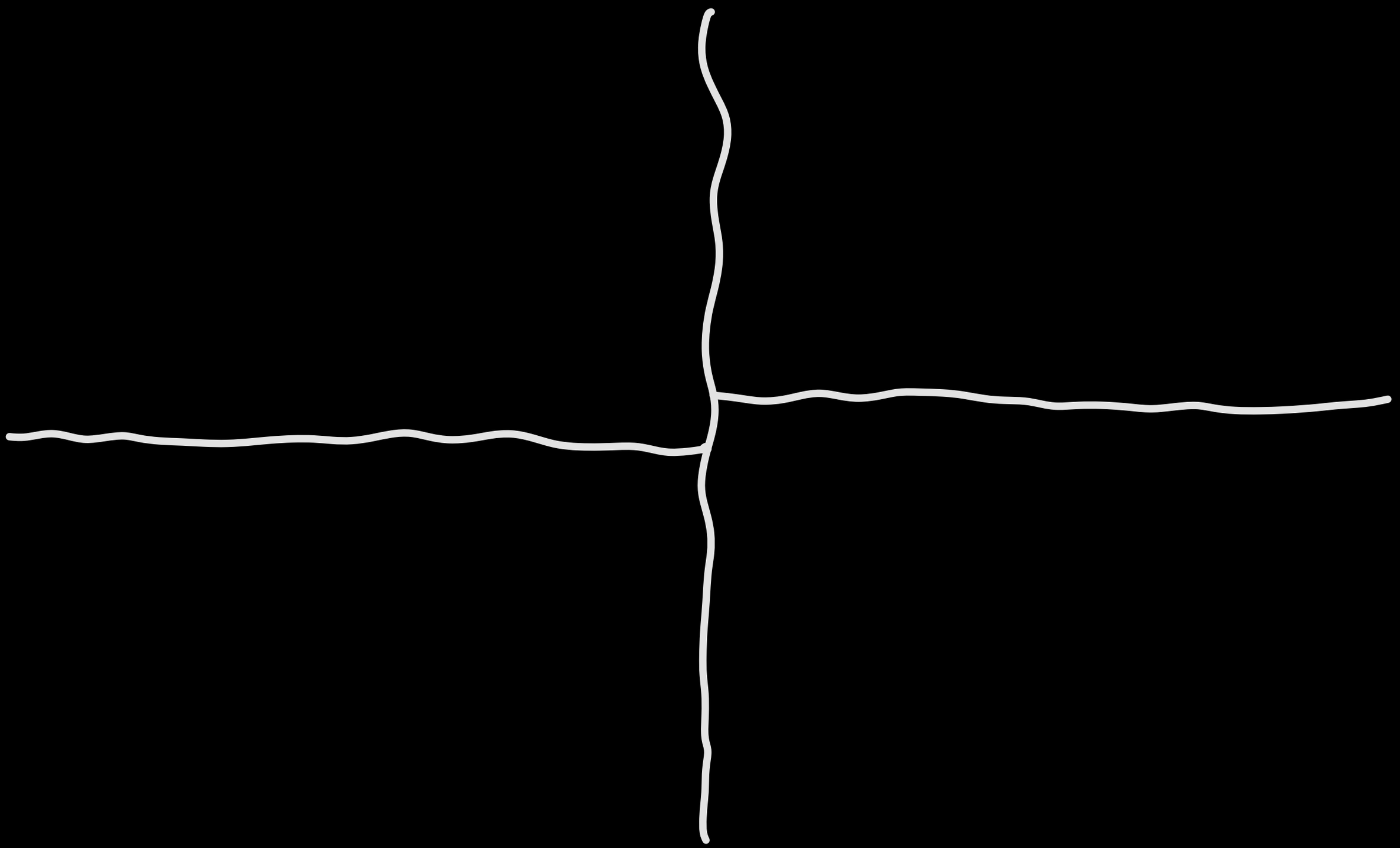
Proof Trees = a bunch of tilings, related by strategies that are sufficient to derive enumerations, generating functions, etc

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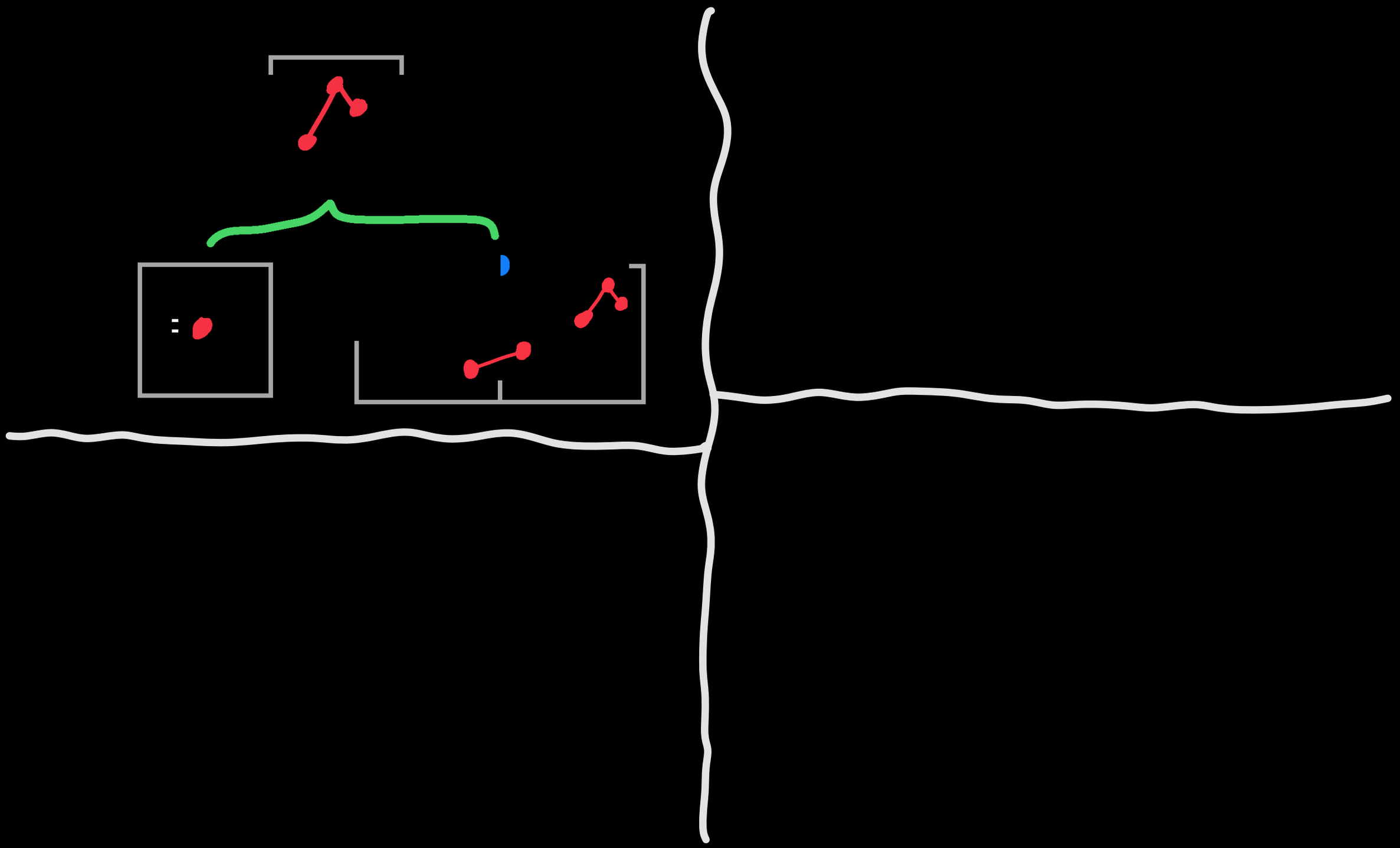
Combinatorial Exploration = repeatedly apply strategies to tilings until you find a proof tree


dN
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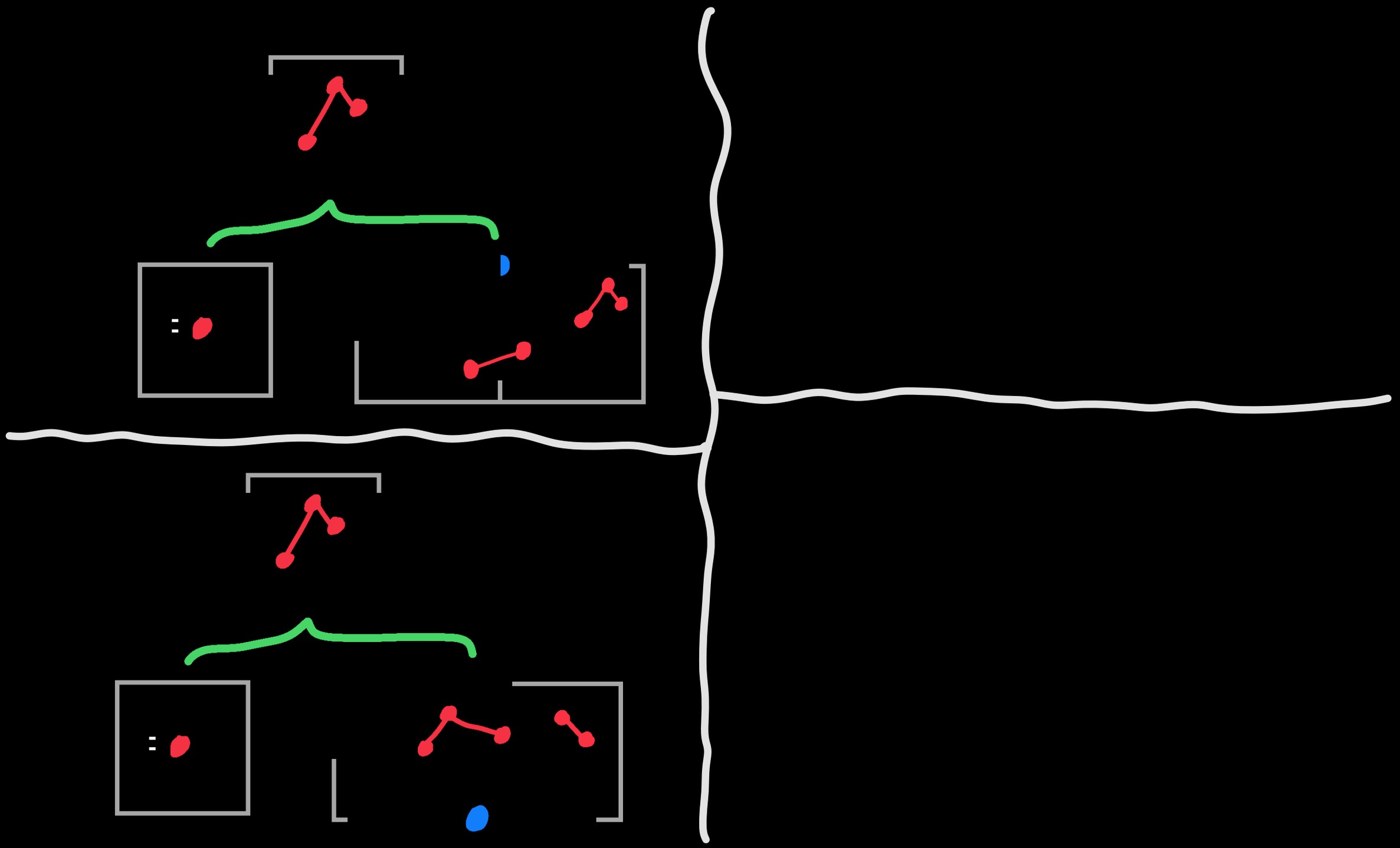
Point Placement



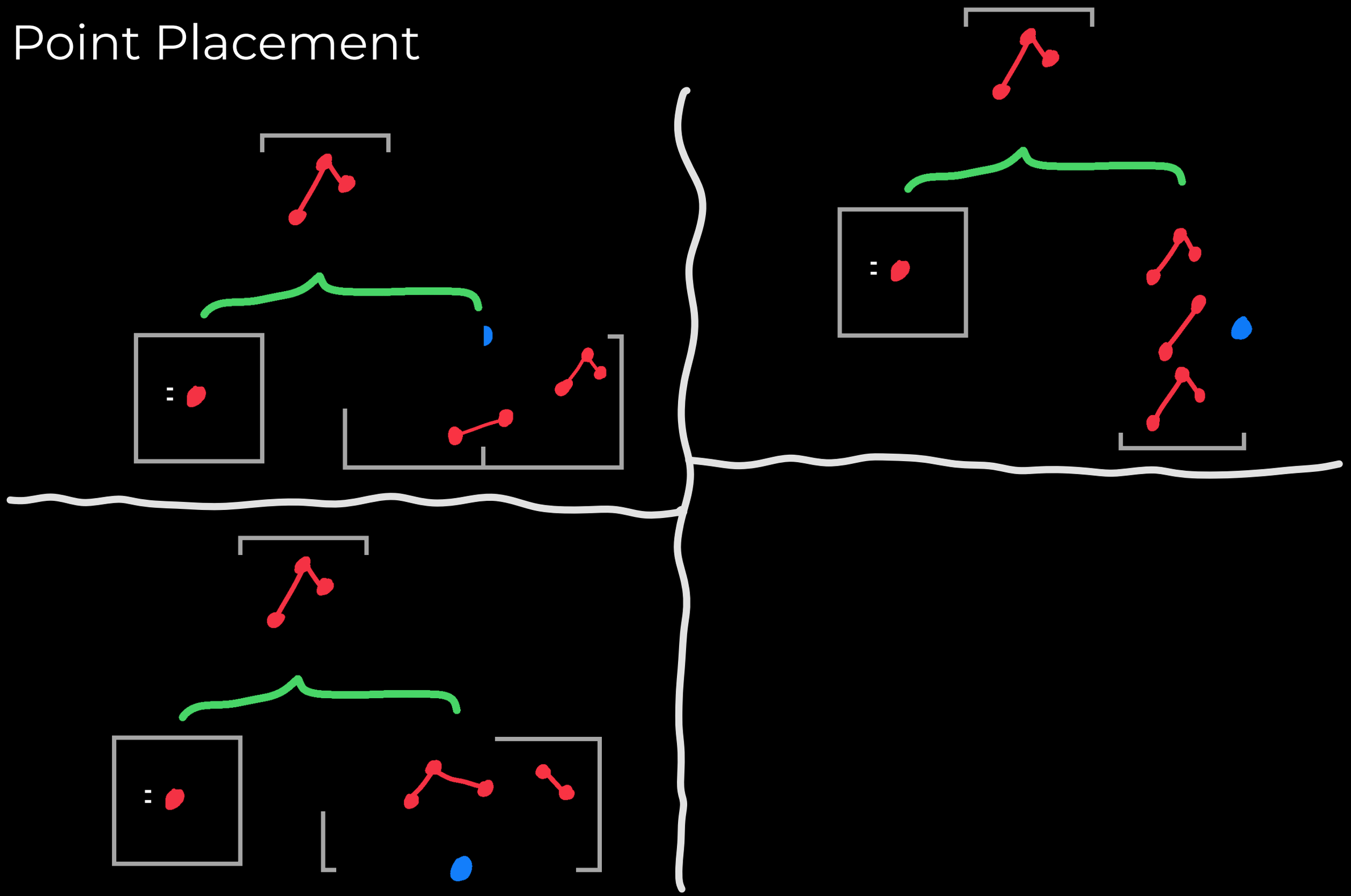
Point Placement



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Point Placement



Point Placement

